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Chapter 6

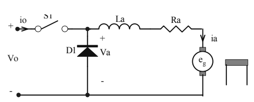
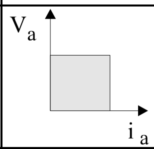
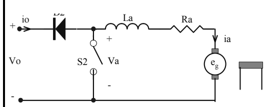
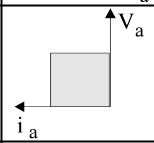
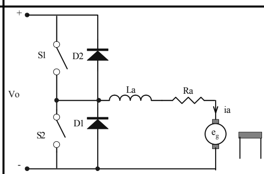
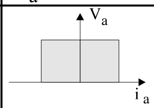
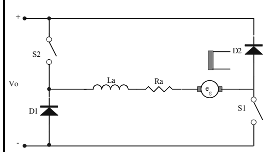
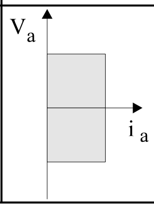
CHOPPER-CONTROLLED DC BRUSH MOTOR DRIVES

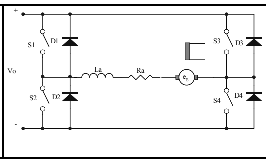
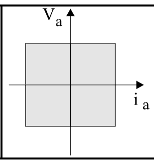
6.1. INTRODUCTION

The DC chopper is a DC to DC power electronic converter (PEC) with forced commutation. It is used for armature voltage control in DC brush motor drives. DC sources to supply DC choppers are batteries or diode rectifiers with output filters so typical for urban electric transportation systems or to low power DC brush motor drives. Thyristors, bipolar power transistors, MOSFETs or IGBTs are used in DC choppers.

The basic configurations are shown in Table 6.1 and they correspond to single, two- or four-quadrant operation.

Table 6.1. Single-phase chopper configurations for the DC brush motors

Type	Chopper configuration	ea-Ia characteristics	Function
First-quadrant (step-down) choppers			Va = V0 for S1 on Va = 0 for S1 off and D1 on
Second quadrant, regeneration (step-up) chopper			Va = 0 for S2 on Va = V0 for S2 off and D2 on
Two quadrant chopper			ea = e0 for S1 or D2 on ea = e0 for S2 or D1 on ia>0 for S1 or D1 on ia<0 for S2 or D2 on
Two quadrant chopper			Va = +V0 for S1 & S2 on Va = –V0 for S1 & S2 off and D1 & D2 on

Four quadrant chopper			S4 on & S3 off S1 & S2 operated Va>0 ia - reversible S2 on & S1 off S3 & S4 operated Va<0 ia - reversible

The first-quadrant chopper (Figure 6.1) is operated by turning on the PES for the interval t_{on}, when the supply voltage is connected to the load. During the interval t_{off}, when the main switch is off, the load current flows through the freewheeling diode D_1. The output voltage e_a is shown in Figure 6.1.

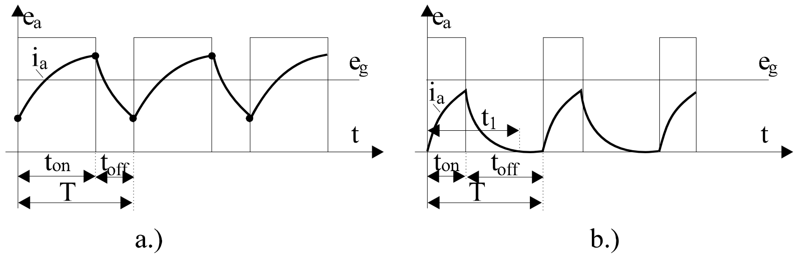


Figure 6.1. First-quadrant chopper operation

a.) continuous mode; b.) discontinuous mode

The average voltage V_{av} is

$$V_{av} = e_a \cdot \frac{t_{on}}{T} \le V_0 \tag{6.1}$$

That is, a step-down chopper.

Constant frequency (constant T) control is preferred in order to improve the input filter operation and reduce the possibility of discontinuous current mode (Figure 6.1b) operation.

The voltage equation for constant speed is

$$V_0 = R_a \cdot i_a + L_a \cdot \frac{di_a}{dt} + e_g; \quad e_g = K_e \cdot \lambda_p \cdot n \quad \text{for } 0 \le t \le t_{on} \tag{6.2}$$

$$0 = R_a \cdot i_a + L_a \cdot \frac{di_a}{dt} + e_g; \quad t_{on} \le t \le t_1; \quad i_a(t_1) = 0,$$

$$t_1 < T \text{ for discontiuous mode} \tag{6.3}$$

For continuous current mode $t_1 = T$ and $i_a(T) = i_a(0) \neq 0$ for steady state.

For the DC brush series motor

$$e_g = K_{ei} \cdot i_a \cdot n + K_{rem} \cdot n \tag{6.4}$$

In (6.4) K_{rem} refers to the remnant flux while the magnetization curve of the machine is considered linear.

The average output voltage for the discontinuous mode may be determined noting that the motor voltage is then zero

$$V_{av} = V_0 \frac{t_{on}}{T} + e_g \cdot \frac{T - t_1}{T}; \qquad t_1 \le T \tag{6.5}$$

The output current expressions are obtained from (6.2)-(6.3)

$$i_a = A \cdot e^{-t \cdot \frac{R_a}{L_a}} + \frac{V_0 - e_g}{R_a}; \qquad \text{for } 0 \le t \le t_{on} \tag{6.6}$$

$$i_a' = A' \cdot e^{-(t - t_{on}) \cdot \frac{R_a}{L_a}} - \frac{e_g}{R_a}; \text{ for } 0 \le t \le t_1 \begin{cases} t_1 = T \text{ for continuous current} \\ t_1 < T \text{ for discontinuous current} \end{cases} \tag{6.7}$$

The continuity condition is

$$i_a(t_{on}) = i_a'(t_{on}) \tag{6.8}$$

The average output current i_{av} is

$$i_{av} = \frac{\int_0^{t_{on}} i_a dt + \int_{t_{on}}^{t_1} i_a' dt}{T} \tag{6.9}$$

For the second-quadrant chopper (Table 6.1b) the DC motor e.m.f. e_g with S_2 on produces a current rise in inductance L_a

$$R_a \cdot i_a + L_a \cdot \frac{di_a}{dt} = -e_g; \qquad \text{for } 0 \le t \le t_{on}; \quad i_a(0) = 0 \tag{6.10}$$

When S_2 is turned off, the energy stored in the inductor is sent back to the source as long as $V_0 > V_a$

$$V_0 - e_g = -R_a \cdot i_a' - L_a \cdot \frac{di_a'}{dt}; \quad t_{on} \le t \le T \tag{6.11}$$

with the solution

$$i_a = -\frac{e_g}{R_a} + B \cdot e^{-t \cdot \frac{R_a}{L_a}} + i_{a0} \tag{6.12}$$

$$i_a' = +\frac{V_0 - e_g}{R_a} + B' \cdot e^{-(t - t_{on}) \cdot \frac{R_a}{L_a}} \tag{6.13}$$

The boundary conditions are

$$i_a(t_{on}) = i_a'(t_{on}), i_a(0) = i_{a0} \text{ and } i_a'(T) = i_{a0} \tag{6.14}$$

It is thus possible with $e_g<V_0$ to retrieve the energy back from the DC brush motor by using the inductor L_a as an energy sink (Figure 6.2).

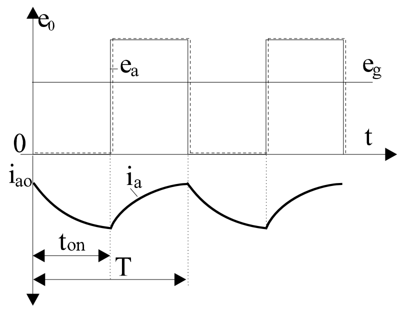


Figure 6.2. Second-quadrant chopper operation

The two-quadrant chopper (Table 6.1c,d) is a combination of one first and one second-quadrant chopper. Finally two-quadrant choppers are combined to obtain a four-quadrant chopper.

As the chopper is an on-off switch, the source current is chopped (Figure 6.3). This makes the peak input power demand high. Also, the supply current (Figure 6.3) has harmonics which produce voltage fluctuations, signal interference, etc.

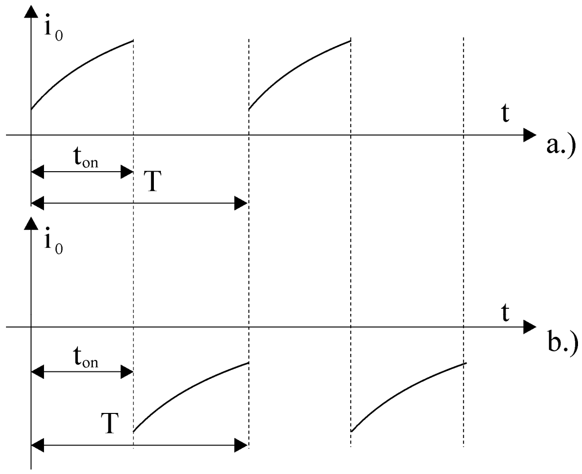


Figure 6.3. Source current waveforms

a.) first-quadrant operation, b.) second-quadrant operation.

An LC input filter (Figure 6.4) will provide a path for the ripple current such that only (approximately) the average current is drawn from the supply.

The n^{th} harmonic current i_n in the supply (Figure 6.4b) is

$$i_n = \frac{X_c / n}{\left(nX_L - X_C / n\right)} I_{sn} = \frac{I_{sn}}{\left(nf_{ch} / f_r\right)^2 - 1} \tag{6.15}$$

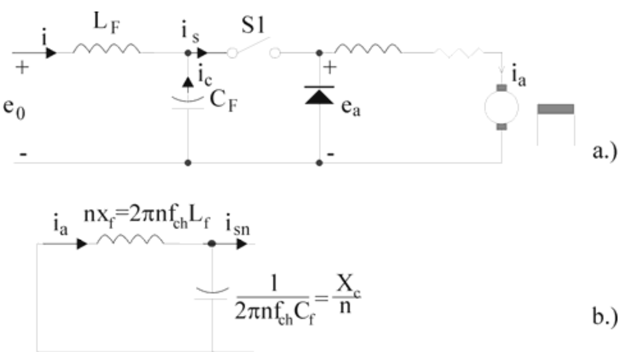


Figure 6.4. First-quadrant chopper with LC input filter

a.) basic circuit, b.) equivalent circuit for nth harmonic

where f_{ch} is the chopping frequency (f_{ch} = 1 / T) and f_r is the resonance frequency of the filter $f_r = 1/2\pi\sqrt{LC}$. To avoid resonance $f_{ch} \geq (2-3)f_r$. Given the variety of configurations in Table 6.1 we will proceed directly with numerical examples in parallel with theoretical development to facilitate quick assimilation of knowledge.

6.2. THE FIRST-QUADRANT (STEP-DOWN) CHOPPER

A DC brush motor with permanent magnet excitation with the data R_a = 1Ω, $K_e\lambda_p$ = 0.055 Wb / rpm, is fed through a first-quadrant chopper (Table 6.1a) from a 120 VDC supply at a constant (ideal) armature current of 10A.

Determine:

a. the range of duty cycle α from zero to maximum speed;

b. the range of speed.

<u>Solution</u>:

a. The average output voltage V_a is

$$V_a = V_0 \cdot \alpha = 120\alpha \tag{6.16}$$

At standstill n = 0 and thus

$$V_a = R_a i_a = 1 \cdot 10 = 10\text{V}$$

$$\text{Thus} \quad \alpha_{min} = \frac{(V_a)_{n=0}}{V_0} = \frac{10}{120} = \frac{1}{12} \tag{6.17}$$

For maximum speed, the voltage is 120V (α = 1). Consequently, α varies from 1/12 to 1.

b. The voltage equation for maximum speed is

$$V_{a_{max}} = R_a I_a + K_e \lambda_p n_{max} \tag{6.18}$$

$$n_{max} = \frac{120 - 1 \cdot 10}{0.055} = 2000 \text{ rpm} \tag{6.19}$$

The speed range is thus from zero to 2000 rpm.

c. For the DC brush motor and chopper as above and α = 0.3, calculate the actual armature current waveform, its average value, and voltage average value at n = 1600 rpm, for the chopping frequency f_{ch} = 50Hz. Determine the chopping frequency for which the limit between discontinuous and continuous current is reached at same t_{on} as above.

<u>Solution</u>:

We now apply the armature current expressions (6.6)-(6.7) first for f_{ch} = 50Hz.

The turn-on time interval $$t_{on} = \frac{1}{f_{ch}} \cdot \alpha = \frac{0.3}{50} = 6 \cdot 10^{-3}\text{s} = 6 \text{ ms} \tag{6.20}$$

with $T = \frac{1}{f_{ch}} = \frac{1}{50} = 0.02\text{s} = 20$ ms

$$i_a = A \cdot e^{-t\frac{1}{5\cdot 10^{-3}}} + \frac{120 - 0.055 \cdot 1600}{1} = A \cdot e^{-200t} + 32$$

$$i_a' = A' e^{-200\left(t - 6\cdot 10^{-3}\right)} - \frac{88}{1} \tag{6.21}$$

Assuming discontinuous current mode

$$\left(i_a\right)_{t=0} = 0; \quad A = -32 \tag{6.22}$$

$$\left(i_a'\right)_{t_{on}} = \left(i_a\right)_{t_{on}}; \qquad A' - 88 = 32\left(1 - e^{-200\cdot 6\cdot 10^{-3}}\right) = 22.36$$

Also

$$A' = 110.36 \tag{6.23}$$

The current i_a' becomes zero at $t = t_1$

$$A' e^{-200\left(t_1 - 6\cdot 10^{-3}\right)} - 88 = 0; \; 110.36 \cdot e^{-200\left(t_1 - 6\cdot 10^{-3}\right)} - 88 = 0$$

$$t_1 = 7.132 \cdot 10^{-3}\,\text{s} < T = 20\,\text{ms} \tag{6.24}$$

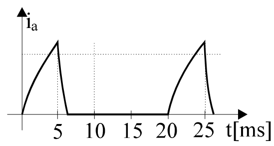


Figure 6.5. Discontinuous current

Thus indeed the current is discontinuous.
The average current i_{av} is:

$$i_{av} = \frac{1}{T}\left[\int_0^{t_{on}} i_a dt + \int_{t_{on}}^{t_1} i_a' dt\right] =$$

$$\frac{1}{2\cdot 10^{-2}}\left[32\int_0^{6\cdot 10^{-3}}\left(1 - e^{-200t}\right)dt + \int_{6\cdot 10^{-3}}^{7.132\cdot 10^{-3}}\left(110.36 \cdot e^{-200\left(t - 6\cdot 10^{-3}\right)} - 88\right)dt\right]$$

$$i_{av} = 7.8452\,\text{A} \tag{6.25}$$

The chopping frequency for the limit between the discontinuous and continuous current is obtained for

$$t_1 = T_c = 7.132 \cdot 10^{-3}; \quad f_{ch}' = \frac{1}{T_c} = \frac{1}{7.132 \cdot 10^{-3}} = 140.2\ \text{Hz} \tag{6.26}$$

In this case α becomes

$$\alpha_c = \frac{t_{on}}{T_c} = \frac{6 \cdot 10^{-3}}{7.132 \cdot 10^{-3}} = 0.8412 \tag{6.27}$$

As the current is discontinuous the average voltage is from (6.5)

$$V_{av} = V_0 \frac{t_{on}}{T} + e_g \frac{(T - t_1)}{T} = 120 \cdot 0.3 + 88 \cdot \frac{20 - 7.132}{20}$$
$$= 40 + 56.619 = 96.619 \text{ V} \tag{6.28}$$

6.3. THE SECOND-QUADRANT (STEP-UP) CHOPPER FOR GENERATOR BRAKING

A DC brush motor with PM excitation is fed through a second-quadrant chopper for regenerative braking (Table 6.1b).

The motor data are R_a = 1Ω; L_a = 20mH; e_g = 80V (given speed). The supply voltage V_0 is 120VDC and $t_{on} = 5.10^{-3}$ s.

Determine:

a. The waveform of motor current for zero initial current
b. The waveform of source current
c. The maximum average power generated

Solution:

a. The current waveforms are as shown in Figure 6.2 and 6.3 with i_{a0} = 0.

$$i_a = -\frac{e_g}{R_a} + B \cdot e^{-t\frac{R_a}{L_a}}; \quad 0 < t < t_{on} \tag{6.29}$$

b.

$$i_a' = \frac{V_0 - e_g}{R_a} + B' e^{-(t - t_{on})\frac{R_a}{L_a}}; \quad t_{on} < t < T \tag{6.30}$$

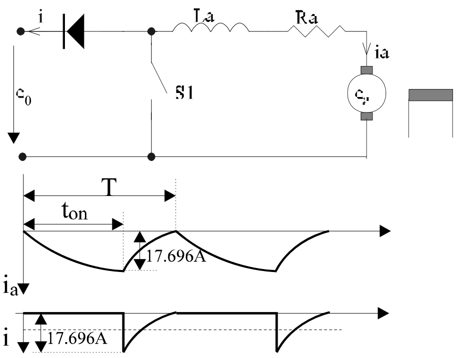


Figure 6.6. The step-up second-quadrant chopper.

The boundary conditions are

$$(i_a)_{t=0} = 0; \quad (i_a)_{t=t_{on}} = (i_a')_{t=t_{on}}; \quad (i_a')_{t=T} = 0 \tag{6.31}$$

The unknowns are B, B′ and T. Consequently,

$$B = \frac{e_g}{R_a} = \frac{80}{1} = 80 \tag{6.32}$$

$$80 \cdot e^{-5 \cdot 10^{-3} \cdot 50} - 80 = \frac{(120 - 80)}{1} + B'; \quad B' = -57.696 \tag{6.33}$$

$$40 - 57.696 \cdot e^{-(T - t_{on}) \cdot 50} = 0; \quad T - t_{on} = 7.326 \cdot 10^{-3}\,s$$

$$\text{thus } T = (7.326 + 5) \cdot 10^{-3}\,s = 12.326 \cdot 10^{-3}\,s \tag{6.34}$$

Note that the source current occurs during the S_1 turn-off and is negative, proving the regenerative operation.

The average source current i_{av} is

$$i_{av}' = \frac{1}{T}\int_{t_{on}}^{T} i_a' dt = \frac{1}{T}\left[\frac{e_0 - e_g}{R_a}(T - t_{on}) - \frac{L_a}{R_a}B'\left(e^{-(T - t_{on})\frac{R_a}{L_a}} - 1\right)\right] =$$

$$= \frac{1}{12.326 \cdot 10^{-3}}\left[\frac{40}{1} \cdot 7.326 \cdot 10^{-3} + 20 \cdot 10^{-3} \cdot 57.696 \cdot \left(e^{-7.326 \cdot 10^{-3} \cdot 50} - 1\right)\right];$$

$$i_{av}' = -4.938\ A \tag{6.35}$$

The average power regenerated P_{av} is

$$P_{av} = -i_{av}' \cdot V_a = 4.938 \cdot 120 = 592.56\ W \tag{6.36}$$

6.4. THE TWO-QUADRANT CHOPPER

Consider a two-quadrant chopper (Figure 6.7) supplying a DC brush motor with separate excitation. The load current varies between $I_{max}>0$ and $I_{min} < 0$.

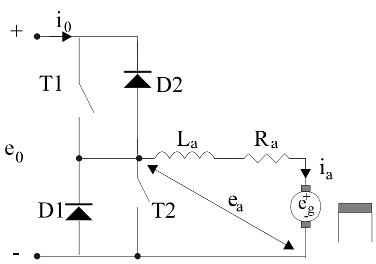


Figure 6.7. Two-quadrant chopper supplying a separately excited DC brush motor

Determine:

a. The voltage and current waveforms for the load current varying between an $I_{max}>0$ and $I_{min}<0$ with $I_{max} > |I_{min}|$.

b. Derive the expression of the conducting times t_{d2} and t_{d1} of the diodes D_1 and D_2 for case a.

Solution:

a. Let us first draw the load current which varies from a positive maximum to a negative minimum (Figure 6.8). The conduction intervals for each of the four switches T_1, D_2, D_1, T_2 are

$$\begin{aligned} &T_1 \text{ for } t_{d2} < t < t_c; \\ &T_2 \text{ for } t_{d1} < t < T; \\ &D_1 \text{ for } t_c < t < t_{d1}; \\ &D_2 \text{ for } 0 < t < t_{d2} \end{aligned} \tag{6.37}$$

b. The equations for the current are:

$$V_0 - e_g = R_a i_a + L_a \frac{di_a}{dt}; \quad \text{for } 0 < t < t_c \tag{6.38}$$

$$-e_g = R_a i_a + L_a \frac{di_a}{dt}; \quad \text{for } t_c < t < T \tag{6.39}$$

with the solutions:

$$i_a = \frac{V_0 - e_g}{R_a} + A \cdot e^{-t\frac{R_a}{L_a}}; \quad 0 < t \le t_c \tag{6.40}$$

$$i_a' = -\frac{e_g}{R_a} + A' \cdot e^{-(t-t_c)\frac{R_a}{L_a}}; \quad t_c < t \le T \tag{6.41}$$

with the boundary conditions $i_a(0) = i_{min}$, $i_a'(T) = i_{min}$ and $i_a(t_c) = i_a'(t_c)$.

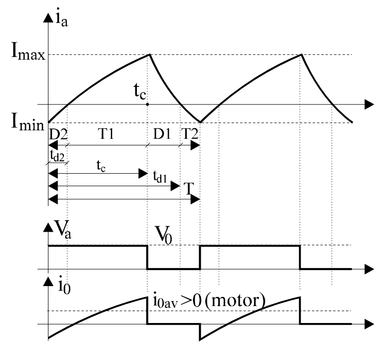


Figure 6.8. Voltage and current waveforms of two-quadrant chopper-fed DC motor

The unknowns A, A′ and t_c are obtained from

$$A = I_{min} - \frac{e_0 - e_g}{R_a} \tag{6.42}$$

$$A' = I_{max} + \frac{e_g}{R_a} \tag{6.43}$$

$$T - t_c = -\frac{L_a}{R_a} \cdot \ln\left[\left(A' - \frac{e_0}{R_a}\right) / A\right] \tag{6.44}$$

Consider a DC brush motor whose data are e_0 = 120V, R_a = 1Ω, L_a = 5mH, e_g = 80V, I_{max} = 5A, I_{min} = –2A and chopping frequency f_{ch} = 0.5kHz. For the two-quadrant chopper as above calculate:

a. The $t_c / T = \alpha_{on}$ ratio.
b. The conducting intervals of the 4 switches.

<u>Solution</u>:

a. The constants A, A′, t_c expressions developed above ((6.42)-(6.44)) yield

$$A = I_{min} - \frac{e_0 - e_g}{R_a} = -2 - \frac{120 - 80}{1} = -42 \text{ A} \tag{6.45}$$

$$A' = I_{max} + \frac{e_g}{R_a} = 5 + \frac{80}{1} = 85 \text{ A} \tag{6.46}$$

$$T = \frac{1}{f_{ch}} = \frac{1}{500} s = 0.002 s = 2 \text{ ms} \tag{6.47}$$

$$T - t_c = -\frac{L_a}{R_a} \cdot \ln\left[\left(A' - \frac{e_0}{R_a}\right) / A\right] =$$

$$-\frac{5 \cdot 10^{-3}}{1} \cdot \ln\left[\left(85 - \frac{120}{1}\right) / (-42)\right] = 0.9116 \cdot 10^{-3} s$$

$$t_c = 2 \cdot 10^{-3} - 0.5116 \cdot 10^{-3} = 1.0884 \cdot 10^{-3} s \tag{6.48}$$

b. The conducting interval of D_2, t_{d2}, corresponds to $i_a = 0$

$$\frac{V_0 - e_g}{R_a} + A \cdot e^{-t_{d2}\frac{R_a}{L_a}} = 0$$

$$t_{d2} = --\frac{L_a}{R_a} \cdot \ln\left[\frac{V_0 - e_g}{-A \cdot R_a}\right] = \frac{-5 \cdot 10^{-3}}{1} \ln\left(\frac{120 - 80}{-(-42) \cdot 1}\right) = 0.2439 \cdot 10^{-3} s \tag{6.49}$$

Thus the main switch T_1 conducts for a time interval

$$t_c - t_{d2} = (1.0884 - 0.2439)\cdot 10^{-3} = 0.84445\cdot 10^{-3}\,s \tag{6.50}$$

To calculate the conducting time of the diode D_1 we apply the condition $i_a'(t_{d1}) = 0$

$$\frac{-e_g}{R_a} + A'\cdot e^{-(t_{d1}-t_c)\frac{R_a}{L_a}} = 0 \tag{6.51}$$

$$t_{d1} - t_c = -\frac{L_a}{R_a}\cdot \ln\left[\frac{e_g}{A'\cdot R_a}\right] = \frac{-5\cdot 10^{-3}}{1}\ln\left(\frac{80}{85\cdot 1}\right) = 0.303\cdot 10^{-3}\,s \tag{6.52}$$

Consequently, the diode D_1 conducts for 0.303 ms. Finally, the static switch T_2 conducts for the time interval

$$T - t_{d1} = (2 - 1.0884 - 0.303)\cdot 10^{-3} = 0.6084\cdot 10^{-3}\,s \tag{6.53}$$

Note: As seen above, the two-quadrant operation of the chopper resides in the variation of t_c / T as the main switches command signals last t_c and, respectively, T–t_c intervals though they conduct less time than that, allowing the diodes D_2 and D_1 to conduct. The two-quadrant chopper has the advantage of natural (continuous) transition from motor to generator action.

6.5. THE FOUR-QUADRANT CHOPPER

A DC brush motor with separate excitation is fed through a four-quadrant chopper (Table 6.1e). Show the waveforms of voltage and current in the third and fourth quadrants.

Solution:

The basic circuit of a four-quadrant chopper is shown in Figure 6.9.

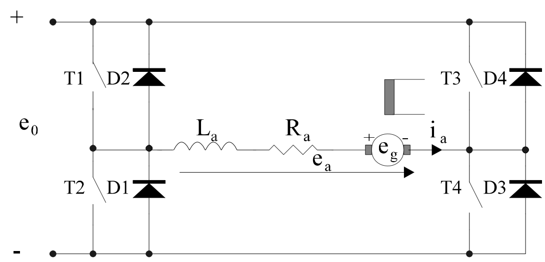


Figure 6.9. DC brush motor fed through a four-quadrant chopper

If T_4 is on all the time, T_1-D_1 and T2-D2 provide first- and (respectively) second-quadrant operations as shown in previous paragraphs. With T_2 on all the time and T_3-D_3 and, respectively, T_4-D_4 the third- and fourth-quadrant operations is obtained (Figure 6.10). So, in fact, we have 2 two-quadrant choppers acting in turns.

However, only 2 out of 4 main switches are turned on and off with the frequency f_{ch} while the third main switch is kept on all the time and the fourth one is off all the time.

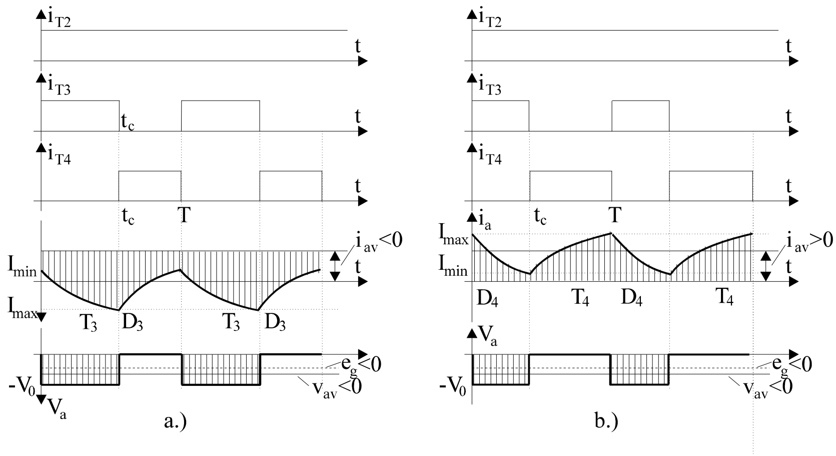


Figure 6.10. Four-quadrant chopper supplying a DC brush motor

a.) Third quadrant: iav<0, Vav<0; b.) Fourth quadrant: iav>0, Vav<0.

Four-quadrant operation is required for fast response reversible variable speed drives.

As expected, discontinuous current mode is also possible but it should be avoided by increasing the switching frequency f_{ch} or adding an inductance in series with the motor.

Let us assume that:

A DC brush motor, fed through a four-quadrant chopper, works as a motor in the third quadrant (reverse motion). The main data are V_0 = 120V, R_a = 0.5Ω, L_a = 2.5mH, rated current I_{an} = 20A; rated speed n_n = 3000 rpm; separate excitation.

a. Calculate the rated e.m.f., e_g, and rated electromagnetic torque, T_e.

b. For n = –1200 rpm and rated average current ($i_{av} = -I_{an}$) determine the average voltage V_{av}, $t_c / T = \alpha_{on}$, and maximum and minimum values of motor current I_{max} and I_{min} for 1kHz switching frequency.

<u>Solution</u>:

a. The motor voltage equation for steady state is:

$$V_{av} = R_a i_a + e_g \tag{6.54}$$

for rated values $V_{av} = V_0$ = 120V, $i_a = i_{an}$ = 20A, thus

$$e_{gn} = K_e \lambda_p n_n = V_{av} - R_a i_a = 120 - 20 \cdot 0.5 = 110 \text{ V} \tag{6.55}$$

$$K_e\lambda_p = \frac{e_{gn}}{n_n} = \frac{110}{50} = 2.2 \text{ Wb} \tag{6.56}$$

b. The motor equation in the third quadrant is

$$V_{av} = R_a i_a + e_g = 0.5\cdot(-20) + 2.2\cdot(-20) = -54 \text{ V}, \tag{6.57}$$

the conducting time t_c for T_3 (Figure 6.10a) is

$$\frac{t_c}{T} = \frac{V_{av}}{-V_0} = \frac{-54}{-120} = 0.45 \tag{6.58}$$

$$t_c = T\cdot 0.45 = \frac{1}{f_{ch}}\cdot 0.45 = \frac{1}{10^3}\cdot 0.45 = 0.45\cdot 10^{-3}\,\text{s} \tag{6.59}$$

Form ((6.40)-(6.41)) the motor current variation (Figure 6.10a) is described by

$$i_a = \frac{V_0' - e_g}{R_a} + A\cdot e^{-t\frac{R_a}{L_a}}; \quad 0 < t \le t_c \tag{6.60}$$

$$i_a' = -\frac{e_g}{R_a} + A'\cdot e^{-(t-t_c)\frac{R_a}{L_a}}; \quad t_c < t \le T \tag{6.61}$$

The current continuity condition ($i_a(t_c) = i_a'(t_c)$) provides

$$t_c = -\frac{L_a}{R_a}\cdot \ln\left[\left(A' - \frac{V_0'}{R_a}\right)/A\right] \tag{6.62}$$

The second condition is obtained from the average current expression

$$i_{av} = \frac{1}{T}\left[\int_0^{t_c} i_a dt + \int_{t_c}^{T} i_a' dt\right] =$$

$$\frac{1}{T}\left\{\frac{V_0' - e_g}{R_a} t_c - \frac{e_g}{R_a}(T - t_c) + \frac{L_a}{R_a}\left[\left(1 - e^{-t_c\frac{R_a}{L_a}}\right)A + A'\left(1 - e^{-(T-t_c)\frac{R_a}{L_a}}\right)\right]\right\} \tag{6.63}$$

From (6.62) and (6.63) we obtain:

$$\left(A' - \frac{V_0'}{R_a}\right) / A = e^{-t_c \frac{R_a}{L_a}};$$

$$V_0' = -V_0; \quad e_g = K_e \lambda_p n = 2.2 \cdot (-20) = -44 \text{ V}$$

$$\left(A' + \frac{(-120)}{0.5}\right) / A = e^{-0.45 \cdot 10^{-3} \frac{0.5}{2.5 \cdot 10^{-3}}} = 0.914 \tag{6.64}$$

$$-20 = 10^3 \{ \frac{-120 - (-44)}{0.5} 0.45 \cdot 10^{-3} - \frac{(-44)}{0.5} 0.55 \cdot 10^{-3} + \frac{2.5 \cdot 10^{-3}}{0.5} \left[\left(1 - e^{-0.45 \cdot 10^{-3} \frac{0.5}{2.5 \cdot 10^{-3}}}\right) A + A' \left(1 - e^{-0.55 \cdot 10^{-3} \frac{0.5}{2.5 \cdot 10^{-3}}}\right) \right] \} \tag{6.65}$$

$$-20 = -20 + 0.43A + 0.5205A' \tag{6.66}$$

$$0.43A + 0.5205A' = 0 \tag{6.67}$$

$$A' + 240 = 0.914A \tag{6.68}$$

$$A = 137.62; \; A' = -113.92 \tag{6.69}$$

Now we may calculate $I_{min} = i_a(0)$

$$I_{min} = A + \frac{V_0' - e_g}{R_a} = 137.92 + \frac{-120 - (-44)}{0.5} = -15.08 \text{ A} \tag{6.70}$$

Also $I_{max} = i_a'(t_c)$

$$I_{max} = A' - \frac{e_g}{R_a} = -113.92 + \frac{-(-44)}{0.5} = -25.92 \text{ A} \tag{6.71}$$

6.6. THE INPUT FILTER

A first-quadrant chopper with an L-C input filter supplies a DC brush motor with PM excitation under constant current start-up.

a. Demonstrate that maximum rms ripple current in the chopper current i_{ch} occurs at a duty cycle $\alpha = 0.5$.
b. For $f_{ch} = 400$ Hz, $I_a = 100$ A, the rms fundamental (AC) current allowed in the supply is 10% of DC supply current. Capacitors of 1mF which can take 5 A rms ripple current are available. Determine L_f and C_f of the filter for $f_{ch} > 2f_r$.
c. For case b. calculate the DC, first and third harmonics of the supply current.

Solution:

a. The chopper configuration (Figure 6.11) provides square current pulses for i_s of width α.

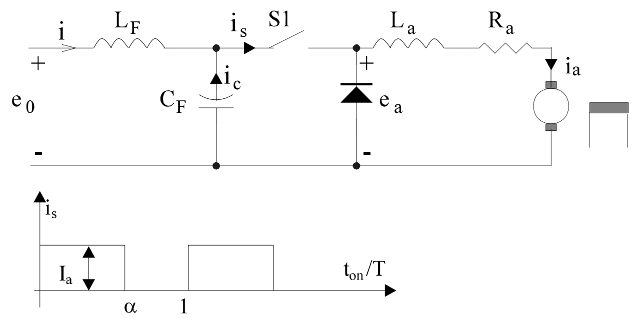


Figure 6.11. First-quadrant chopper with LfCf filter

a.) basic circuit, b.) chopper input current

Thus, the DC, I_{sdc}, rms I_{srms} and ripple $I_{sripple}$ components of chopper currents are:

$$I_{sdc} = I_a \cdot \alpha \tag{6.72}$$

$$I_{srms} = \sqrt{\int_0^{\alpha} I_a^{\,2} d\alpha} = I_a\sqrt{\alpha} \tag{6.73}$$

$$I_{sripple} = \sqrt{\left(I_a\sqrt{\alpha}\right)^2 - \left(I_a\alpha\right)^2} = I_a\sqrt{\left(\alpha - \alpha^2\right)} \tag{6.74}$$

The maximum ripple current is obtained for $\frac{dI_{sripple}}{d\alpha} = 0$ which leads to $\alpha = 1 / 2$. The filter design will be performed for this worst case.

b. To design the filter we need first the chopper current harmonics content which, for $\alpha = 0.5$, is

$$i_s = I_{sdc} + \frac{4}{\pi} \cdot \frac{I_a}{2n}\left(\sin\omega t + \sin 3\omega t + \sin 5\omega t\right) \tag{6.75}$$

with
$$I_{sdc} = I_a \cdot \alpha = 100 \cdot \frac{1}{2} = 50 \text{ A} \tag{6.76}$$

$$\left(i_{s_1}\right)_{rms} = \frac{4 \cdot 100}{\pi \cdot 2 \cdot 1} \cdot \frac{1}{\sqrt{2}} = 45.1733 \text{ A} \tag{6.77}$$

$$\left(i_{s_3}\right)_{rms} = \frac{4 \cdot 100}{\pi \cdot 2 \cdot 3} \cdot \frac{1}{\sqrt{2}} = 15.05 \text{ A} \tag{6.78}$$

$$\left(i_{s_5}\right)_{rms} = \frac{4 \cdot 100}{\pi \cdot 2 \cdot 5} \cdot \frac{1}{\sqrt{2}} = 9.03 \text{ A} \quad (6.79)$$

The input (source) AC current is not to surpass 10% of DC input current, that is $I_1 = I_{sdc} \cdot \frac{10}{100} = 50 \cdot \frac{10}{100} = 5$ A.

According to Equation (6.15) for n = 1 we obtain

$$I_1 = \frac{x_C}{x_L - x_C} \cdot I_{s_1}; \quad 5 = \frac{x_C}{x_L - x_C} \cdot 45 \quad \text{or} \quad x_L = 10x_C \quad (6.80)$$

Also, the fundamental capacitor current I_{C1} is

$$I_{C1} = \frac{x_L}{x_L - x_C} \cdot I_{s_1} = \frac{10x_C}{10x_C - x_C} \cdot 45 = 50 \text{ A} \quad (6.81)$$

As each 1mF capacitor can take 5A, 10 such capacitors in parallel are needed and thus C_f = 10mF. On the other hand, the reactance x_L is

$$x_L = 10x_C = 10 \cdot \frac{1}{2 \cdot \pi \cdot 400 \cdot 10 \cdot 10^{-3}} = 0.398 \ \Omega \quad (6.82)$$

$$L_f = \frac{x_L}{2 \cdot \pi \cdot 400} = \frac{0.398}{2 \cdot \pi \cdot 400} = 0.15847 \cdot 10^{-3} \text{ H} \quad (6.83)$$

The filter resonance frequency f_r is

$$f_r = \frac{1}{2 \cdot \pi \sqrt{L_f C_f}} = \frac{1}{2 \cdot \pi \sqrt{0.15847 \cdot 10^{-3} \cdot 10^{-2}}} = 127 \text{ Hz} \quad (6.84)$$

The ratio between the chopper switching frequency f_{ch} and the filter resonance frequency f_r is

$$f_{ch} / f_r = 400 / 127 = 3.15$$

c. The same AC current components (6.15) (after filtering) are

$$I_1 = \frac{I_{s_1}}{\left(f_{ch} / f_r\right)^2 - 1} \approx \frac{45}{3.15^2 - 1} = 5 \text{ A} \quad (6.85)$$

$$I_3 = \frac{I_{s_1}}{\left(3f_{ch} / f_r\right)^2 - 1} \approx \frac{15}{(3 \cdot 3.15)^2 - 1} = 0.17 \text{ A} \quad (6.86)$$

$$I_5 = \frac{I_{s_1}}{\left(5f_{ch} / f_r\right)^2 - 1} \approx \frac{9}{(5 \cdot 3.15)^2 - 1} = 0.036 \text{ A} \quad (6.87)$$

As noted the $L_f C_f$ filter produces a drastic reduction of source-current harmonics.

6.7. DIGITAL SIMULATION THROUGH MATLAB/SIMULINK

Simulation results of a DC motor drive with a four-quadrant chc presented. The motor model was integrated in a block (DC brush motor in Figure 6.12). Changing of motor parameters is done by clicking on their block. A dialogue box appears and you can change them by modifying their default values. This motor model block includes the possibility of adding an extra inductance in the motor circuit (L_{add}).

The drive system consists of PI speed controller (Ki = 1, Ti = 0.1), PI torque controller (Ki = 50, Ti = 0.0005) and motor blocks. The study examines the system behavior during starting, load perturbation (at 0.2s) and speed reversal with no load (at 0.5 s) and another load perturbation (at 0.6 s).

The integration step (10 μs) can be modified from the Simulink's *Simulation / Parameters*. The chopper frequency is 20 kHz and this block input is the t_c / T_e ratio.

To find out the structure of each block presented above, unmask it (*Options/Unmask*). Each masked block contains a brief description of that block (inputs / outputs / parameters).

The block diagram of the electric drive system is presented in Figure 6.12.

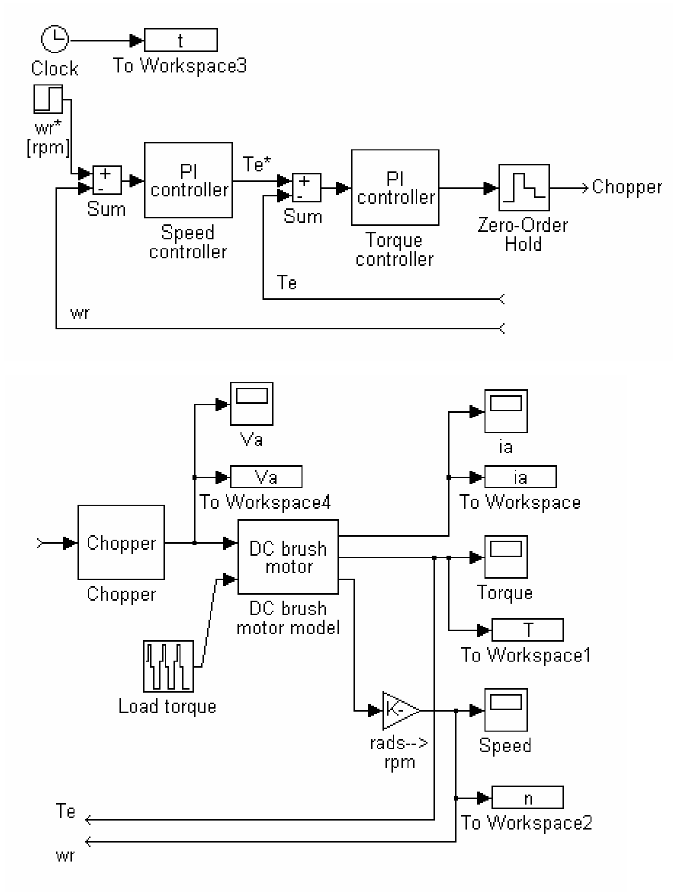


Figure 6.12. DC brush motor drive block diagram

The motor used for this simulation has the following parameters: V_{dc} = 120V, I_n = 20A, n_n = 3000rpm, R_s = 0.5Ω, L_a = 0.0025H, J = 0.001kgm^2, $K_e\lambda_p$ = 2.2Wb.

The Figures (6.13-6.16) represent the speed (Figure 6.13), torque (Figure 6.14) responses and current (Figure 6.15) and voltage (Figure 6.16) waveforms, for the *starting process and load torque (8Nm) applied at 0.2s, and reversal with no load at 0.5s and load torque applied at 0.6s.*

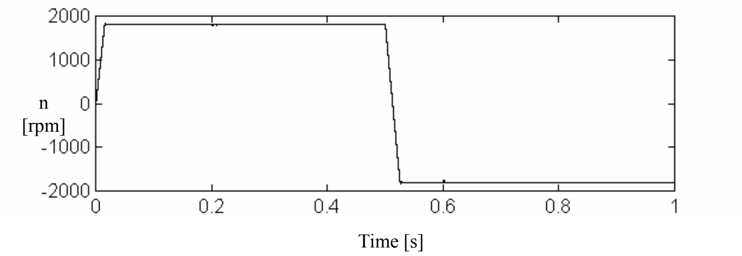


Figure 6.13. Speed transient response

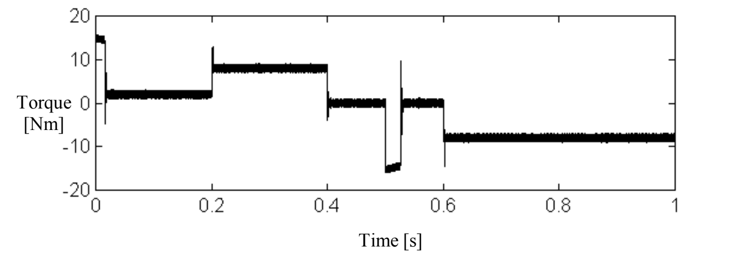


Figure 6.14. Torque response

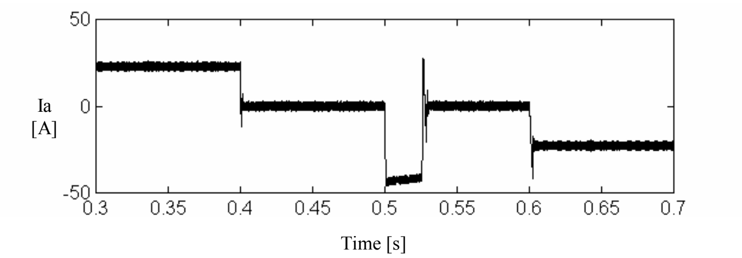


Figure 6.15. Current waveform (ia).

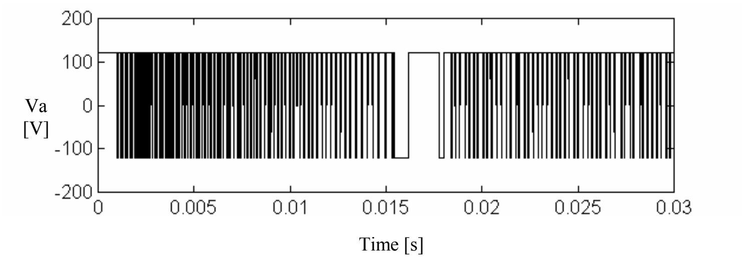


Figure 6.16. Voltage waveform (Va).

Fast response with rather low current ripple is obtained due to the rather high (20 kHz) switching frequency for a 5 ms electrical time constant DC brush PM motor four-quadrant drive.

6.8. SUMMARY

- In this chapter, single, two- or four-quadrant chopper basic configurations in interaction with the DC brush motor (for constant speed) have been introduced.
- In all configurations the PESs have been considered ideal - with instantaneous hard commutation (switching).
- Resonant DC-DC converters have not been treated. The interested reader is invited to study the literature [1].
- Choppers, as rectifiers, face the same problem of continuous-discontinuous current mode switching.
- Under discontinuous current mode, for low voltage low inductance motors and low switching frequency (thyristors), the dependence of output voltage on the on/off time ratios of PESs renders the control sluggish. Increasing the switching frequency or including a series inductance solves the problem.
- The input current has harmonics and thus an input filter is required to reduce them to acceptable limits.
- As the current in the motor pulsates, care must also be exercised in assessing the armature copper losses and the eventual derating of the motor for a certain application when chopper-fed.
- The interaction between the chopper and the motor at constant speed discussed in this chapter should be continued with closed-loop control to produce a variable speed drive. This will be done in the next chapter, though the Matlab simulation paragraph 6.7 anticipates it.

6.9. PROBLEMS

6.1. A step-down (buck) first-quadrant chopper is shown in Figure 6.17.

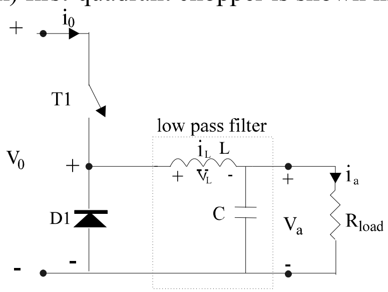


Figure 6.17. Step down (BUCK) chopper

a. Explain the principle and draw the inductance voltage and current waveforms for continuous current mode.
b. Calculate the output voltage and current for a resistive load as a function of duty ratio α, considering ideal switching and zero losses.

6.2. For problem 6.1 find α for the limit between continuous and discontinuous current mode and the average output voltage for discontinuous current mode. Plot the average output voltage versus average output current both for continuous and discontinuous modes.

6.3. A step-up (boost) first-quadrant chopper is shown in Figure 6.18. Considering ideal elements determine:

a. The voltage and current waveforms for continuous current mode.
b. The boundary between continuous and discontinuous current mode.
c. The output voltage ripple for continuous current operation.

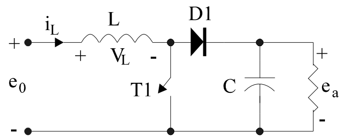


Figure 6.18. Step-up (boost) chopper

6.4. A four-quadrant chopper supplying a DC brush motor with PM excitation is controlled through PWM with bipolar voltage switching, Figure 6.19, where (T_{A+}, T_{B-}) and (T_{A-}, T_{B+}) are controlled simultaneously. Determine:

a. The voltage and current waveforms for positive control voltage $V_{control}$.
b. The voltage and current waveforms for negative control V_{con}.

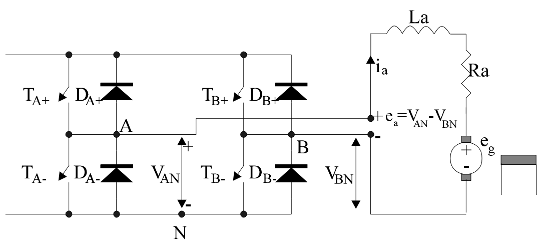


Figure 6.19. Four-quadrant chopper with DC brush motor

6.5. A DC series brush motor has the data: L_{at} = 20 mH, R_{at} = 1 Ω, rated voltage V_n = 120 V, rated current I_{an} = 10 A, rated speed n_n = 1800 rpm. The remnant flux induced voltage at rated speed is e_{grem} = 5V. When fed from a first-quadrant step-down chopper, at 120 Vdc with α = 0.3, f_{ch} = 50 Hz at rated speed, calculate the armature waveform, average current and average voltage.

6.6. Solve problem 6.5. with Matlab/Simulink or PSpice simulation programs.

6.10. SELECTED REFERENCES

1. **N. Mohan, T.M. Undeland, W.P. Robbins**, Power electronics, Second edition, Chapter 7, Wiley, 1995.