Chapter 5

CONTROLLED RECTIFIER DC BRUSH MOTOR DRIVES

5.1. INTRODUCTION

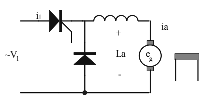
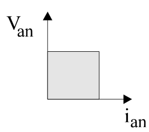
Rectifiers are phase-controlled AC to DC static power converters. The rectifier provides variable DC voltage to the DC brush motor. Thyristors, bipolar transistors, IGBTs or MOSFETs may be used as power electronic switches (PESs) in the converter.

In general, the commutation process is natural, from the DC source, without any additional circuitry. However, for improved power factor forced commutation is used frequently. Phase-controlled power electronic converters are broadly classified as AC-DC single-phase or three phase converters [1]. Their main configurations are shown again in Table 5.1 for convenience (after their classification in Chapter 3).

Half-wave single-phase and semiconverters have one polarity of output voltage e_{av} and current i_{av}. That is, they work in one quadrant. Full converters work in two quadrants: the output voltage e_{av} may be positive or negative while the output current remains positive. Only dual converters can operate in four quadrants.

When the PESs are blocked, the stored energy dissipates through the free wheeling diodes in half-wave and semiconverters. In a single, phase half-wave converter the motor current is discontinuous unless a high additional inductance is added, while for the other converters, the output current may be either continuous or discontinuous. In three-phase converters the motor current is mostly continuous.

Table 5.1. Phase controlled rectifier circuits

Circuit	type	Power range	Ripple frequency	Quadrant operation
	half wave single-phase	below 0.5KW	f_s	 one quadrant

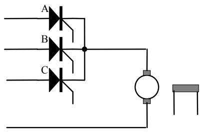
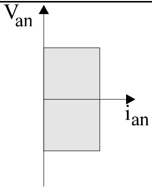
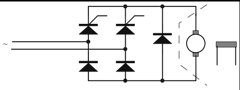
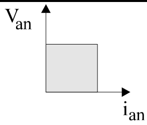
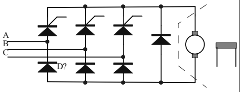
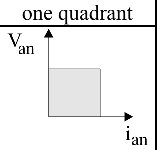
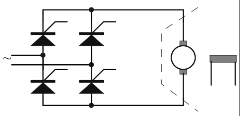
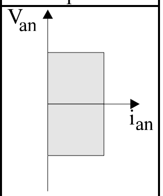
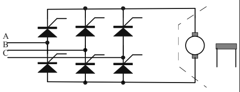
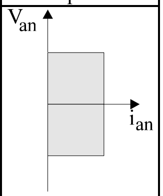
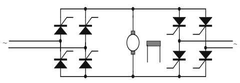
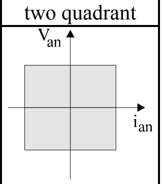
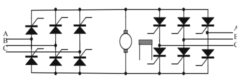
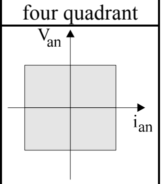
	half wave three-phase	up to 50KW	$3f_s$	 two quadrant

Table 5.1. (continued)

	semi-converter single-phase	up to 75KW	$2f_s$	 one quadrant
	semi-converter three-phase	up to 100KW	$3f_s$	 one quadrant
	full converter single-phase	up to 75KW	$2f_s$	 two quadrant
	full converter three-phase	up to 150KW	$6f_s$	 two quadrant
	Dual converter single-phase	up to 15KW	$2f_s$	 four quadrant
	Dual converter three phase	up to 1500KW	$6f_s$	 four quadrant

5.2. PERFORMANCE INDICES

As the DC motor current, when fed from phase controlled AC-DC converters, is not constant and AC supply current is not sinusoidal, adequate performance indexes for the motor-converter combination should be defined.

The main performance indices related to the motor are:

- the torque-speed characteristic;
- nature of motor current-continuous or discontinuous;
- average motor current I_a:

$$I_a = \frac{1}{T} \cdot \int_{t_1}^{t_1+T} i_a dt \tag{5.1}$$

where: i_a - instantaneous armature current;
T - time period of one cycle of i_a variation.

- Rms motor current I_{ar}

$$I_{ar} = \sqrt{\frac{1}{T} \cdot \int_{t_1}^{t_1+T} {i_a}^2 dt} \tag{5.2}$$

As known the rms current squared is proportional to the heat produced in the armature winding.

- peak motor current i_{ap}; the mechanical commutator stress depends on the peak value of the current.

The main performance related to the input (AC) source are:

- input power factor PF

$$PF = \frac{\text{main input power}}{\text{Rms input volt} \times \text{amperes}} \tag{5.3}$$

If the supply voltage is a pure sinusoid, only the fundamental input current will produce the mean input power and thus

$$PF = \frac{V \cdot I_1 \cdot \cos\varphi_1}{V \cdot I} \tag{5.4}$$

where: V - rms supply phase voltage;
I - rms supply phase current;
I_1 - rms fundamental component of AC supply current;
φ_1 - angle between supply voltage and current fundamentals.

- input displacement factor DF or the fundamental power factor

$$DF = \cos\varphi_1 \tag{5.5}$$

- harmonic factor HF

$$HF = \frac{\sqrt{\left(I^2 - I_1{}^2\right)}}{I_1} = \frac{\sqrt{\sum_{n=2}^{\infty} I_n{}^2}}{I_1} = \frac{I_h}{I_1} \tag{5.6}$$

I_h - rms value of the net harmonic current.

The above performance indices are somewhat similar to those for diode rectifiers (Chapter 3). They are summarized here only for convenience.

The basic motor equations are

$$v_a = R_a \cdot i_a + L_a \cdot \frac{di_a}{dt} + e_g;\ e_g = K_e \cdot \lambda_p \cdot n$$

$$2\pi J \frac{dn}{dt} = T_e - T_{load};\ \ T_e = \frac{e_g \cdot i_a}{2\pi n} = \frac{K_e}{2\pi} \cdot \lambda_p \cdot i_a \tag{5.7}$$

with v_a, i_a motor input voltage and current; e_g - motion induced voltage; R_a, L_a - armature resistance and inductance; J - inertia; T_e - motor torque, T_{load} - load torque. For the steady state, dn/dt = 0, while in general $di_a / dt \neq 0$ as the armature current is not constant in time with phase-controlled converter supplies. To facilitate a quick assimilation of so many controlled rectifier configurations performance we will proceed directly through numerical examples.

5.3. SINGLE PES-CONTROLLED RECTIFIER

A DC brush motor with separate excitation with the data: $K_e\lambda_p$ = 2Wb, $R_a = 5\Omega$, L_a = 0.1H is fed through a thyristor (Figure 5.1) from a single phase AC source whose voltage is $V = V_1 \cdot \sin\omega_1 t = 120\sqrt{2}\sin 376.8t.$ The motor speed is considered constant at n = 750 rpm.

a. Calculate the motor current i_a time variation for a delay angle $\alpha = +30^0$;

b. How does the motor voltage V_a vary in time?

c. How does i_a vary in time in the presence of a free wheeling diode in parallel with the motor armature (Figure 5.1)? The thyristor and the diode are considered as ideal switches.

<u>Solution</u>:

a. For $\omega t > \alpha$ (Figure 5.1) with discontinuous current:

$$V_a(t) = R_a i_a + L_a \frac{di_a}{dt} + K_e \lambda_p n \tag{5.8}$$

with the initial condition $i_a = 0$ for $\omega_1 t = \alpha = \pi/6$.

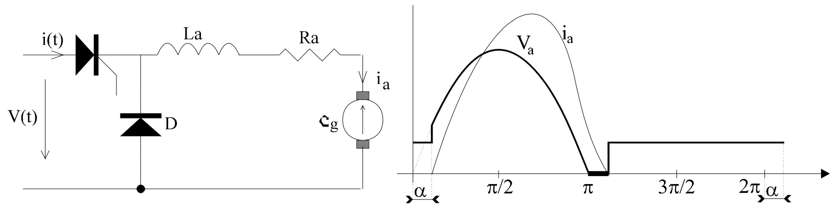


Figure 5.1. A DC brush motor supplied through a thyristor

The steady-state solution is

$$i_{ap} = A + B \cdot \cos\omega_1 t + C \cdot \sin\omega_1 t \tag{5.9}$$

and finally:

$$i_{ap} = -\frac{K_e \cdot \lambda_p \cdot n}{R_a} + \frac{V_1}{\sqrt{R_a^2 + \omega_1^2 \cdot L_a^2}} \cdot \sin(\omega_1 t - \gamma);$$
$$\gamma = \tan^{-1}(\omega_1 L_a / R_a) \tag{5.10}$$

The complete solution:

$$i_a(t) = i_{ap} + A \cdot e^{-tR_a/L_a} + B \tag{5.11}$$

For $t = t_1 = \alpha / \omega_1 = \pi/(6.2\pi.60) = 1.3888.10^{-3}$ s $\qquad i_a(t) = 0$
and thus:

$$\gamma = \tan^{-1}\frac{2 \cdot \pi \cdot 60 \cdot 0.1}{5} = 82.44^0;$$
$$\frac{V_1}{\sqrt{R_a^2 + \omega_1^2 \cdot L_a^2}} = \frac{120}{\sqrt{5^2 + (2 \cdot \pi \cdot 60 \cdot 0.1)^2}} = 3.157 \text{ A} \tag{5.12}$$

and

$$\frac{K_e \cdot \lambda_p \cdot n}{R_a} = \frac{2 \cdot 12.5}{5} = 5 \text{ A} \tag{5.13}$$

$$A = \left[\frac{K_e \cdot \lambda_p \cdot n}{R_a} - \frac{V_1}{\sqrt{R_a^2 + \omega_1^2 \cdot L_a^2}} \cdot \sin(30^0 - \gamma)\right] \cdot e^{-1.3888 \cdot 10^3 \cdot 5/0.1} =$$
$$= [5 - 3.157\sin(-52.44^0)] \cdot 1.07186 = 8.04 \text{ A} \tag{5.14}$$

Finally, $$i_a(t) = 8.04 \cdot e^{-50t} - 5 + 3.157 \cdot \sin(378.4t - 1.43812) \tag{5.15}$$

Equation (5.15) is valid for $\omega_1 t > \pi/6$ until the current becomes zero soon after $\omega_1 t = \pi$, as shown in Figure 5.1.

b. The motor voltage is equal to the source voltage as long as the thyristor is on and becomes equal to minus the e.m.f.: $-e_a = K_e \cdot \lambda_p \cdot n$ = $2 \cdot 12.5 = 25\,\text{V}$, when the motor current is zero (Figure 5.1)

c. In presence of the free wheeling diode D, the latter starts conducting when V(t) becomes negative ($\omega_1 t = \pi$).

From now on, the motor current i_a' flows through the diode until it becomes zero

$$0 = R_a \cdot i_a' + L_a \cdot \frac{di_a'}{dt} + K_e \cdot \lambda_p \cdot n \tag{5.16}$$

with

$$i_a'(0) = i_a(\pi) \tag{5.17}$$

$$i_a' = -\frac{K_e \cdot \lambda_p \cdot n}{R_a} + A' \cdot e^{-t' R_a / L_a} \tag{5.18}$$

$$i_a'(\pi) = i_a(\pi) = \frac{-2 \cdot 12.5}{5} + A' \cdot e^{-50/120} = 3.42729\ \text{A} \tag{5.19}$$

$$A' = 12.78319\ \text{A}.$$

Thus

$$i_a' = -5 + 12.783 \cdot e^{-50t'} \tag{5.20}$$

The current $i_a' = 0$ for $\omega_1 t' = 7.07°$.

The motor voltage V_a becomes zero during the time interval when the free wheeling diode is conducting (180° to 180°+7.07°).

5.4. THE SINGLE-PHASE SEMICONVERTER

A DC motor supplied through a single-phase semiconverter has a constant speed and the motor current is constant in time, I_a = 3A, R_a = 5Ω, $K_e\Phi$ = 2Wb, $v = 120\sqrt{2}\sin 120\pi t$ with $\alpha = \pi/4$.

a. Determine the time waveforms of voltages at motor terminals and along the thyristors and diodes and the corresponding current waveforms.

b. Calculate the rms values of currents through diodes, thyristors and from the AC supply.

c. Determine the motor voltage average value dependence and its maximum value.

d. Calculate the rms of AC supply current fundamental and its phase shift with respect to the AC supply voltage.

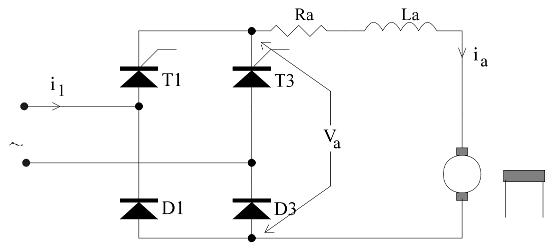
Solution:

a. As the filter inductance L_a is large, the motor current i_a is considered constant (Figure 5.2). The thyristors T_1 and T_3 conduct for positive voltage applied to them along 180°–α regions while the diodes D_1, D_3 conduct for zones of 180°+α degrees (Figure 5.2.d). All currents are rectangular (Figure 5.2d, e) while the motor voltage is either positive or zero (Figure 5.2b).

b. The rms value of thyristor current is

$$I_{T_{1,3}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i_{T_1}{}^2 d(\omega_1 t)} = \sqrt{\frac{1}{2\pi}\int_\alpha^{\pi} i_a{}^2 d(\omega_1 t)} =$$

$$= I_a\sqrt{\frac{\pi-\alpha}{2\pi}} = 3\cdot 0.6123 = 1.837\text{ A} \tag{5.21}$$



a.)

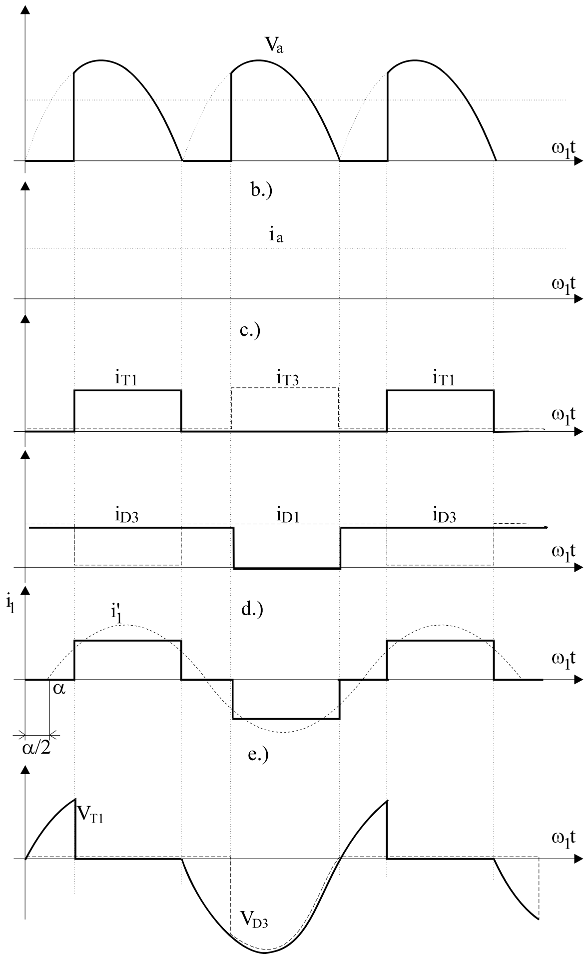


Figure 5.2. DC brush motor supplied through a single-phase semiconverter (with large inductance L_a)

The diode current $I_{D_{1,3}}$

$$I_{D_{1,3}} = \sqrt{\frac{1}{2\pi}\int_0^{\pi+\alpha} i_a^{\,2}\, d(\omega_1 t)} = I_a\sqrt{\frac{\pi+\alpha}{2\pi}} = 3\cdot 0.79 = 2.37\ \text{A} \tag{5.22}$$

The rms primary (supply current)

$$I_1 = \sqrt{\frac{1}{\pi}\int_\alpha^{\pi} i_a^{\,2}\, d(\omega_1 t)} = I_a\sqrt{\frac{\pi-\alpha}{\pi}} = 3\cdot 0.866 = 2.598\ \text{A} \tag{5.23}$$

c. The motor average voltage V_{av} is

$$V_{av} = \frac{1}{\pi}\int_{\alpha}^{\pi} v_a d(\omega_1 t) = \frac{1}{\pi}\int_{\alpha}^{\pi} V\sqrt{2}\sin\omega_1 t \cdot d(\omega_1 t) = \frac{V_1\sqrt{2}}{\pi}(1+\cos\alpha) \quad (5.24)$$

For $\alpha=\pi/4$

$$(V_{av})_{\alpha=\pi/4} = \frac{120\sqrt{2}}{\pi}(1+0.707) = 91.98\,\mathrm{V} \quad (5.25)$$

The maximum average voltage is obtained for $\alpha = 0$;

$$(V_{av})_{\alpha=0} = \frac{120\sqrt{2}}{\pi}(1+1) = 107.77\,\mathrm{V} \quad (5.26)$$

and corresponds to the full bridge diode rectifier (Chapter 3).

d. The AC supply current harmonics $I_{1\nu}$ is

$$I_{1\nu} = \frac{1}{\pi}\int_{0}^{2\pi} i_1 \sin\nu\omega_1 t \cdot d(\omega_1 t) = \frac{4}{\pi\nu} I_a \cos\frac{\nu\alpha}{2} \quad (5.27)$$

The fundamental is:

$$I_{1(1)} = \frac{4}{\pi} 3\cos\frac{\pi}{2} = 3.3313\,\mathrm{A} \quad (5.28)$$

The input displacement power factor DPF = $\cos\varphi_1$ refers to the cosine of the angle between the AC source voltage and current fundamentals. As seen from Figure 5.2b, e $\varphi_1=\alpha/2=\pi/8$. Thus

$$\mathrm{DPF} = \cos\varphi_1 = \cos\pi/8 = 0.9238 \quad (5.29)$$

On the other hand, the power factor PF (5.4) is

$$\mathrm{PF} = \frac{I_{1(1)}\cdot\cos\varphi_1}{\sqrt{2}\cdot I_1} = \frac{3.3313\cdot 0.9238}{1.41\cdot 2.598} = 0.84 \quad (5.30)$$

5.5. THE SINGLE-PHASE FULL CONVERTER

A DC motor of 7kW and 1200 rpm rating with separate excitation is supplied through a single-phase full converter as shown in Figure 5.3a. The other data are R_a = 0.2Ω, rated current I_{ar} = 40A, $K_e\lambda_p$ = 10Wb; the AC supply voltage (rms) is 260V.

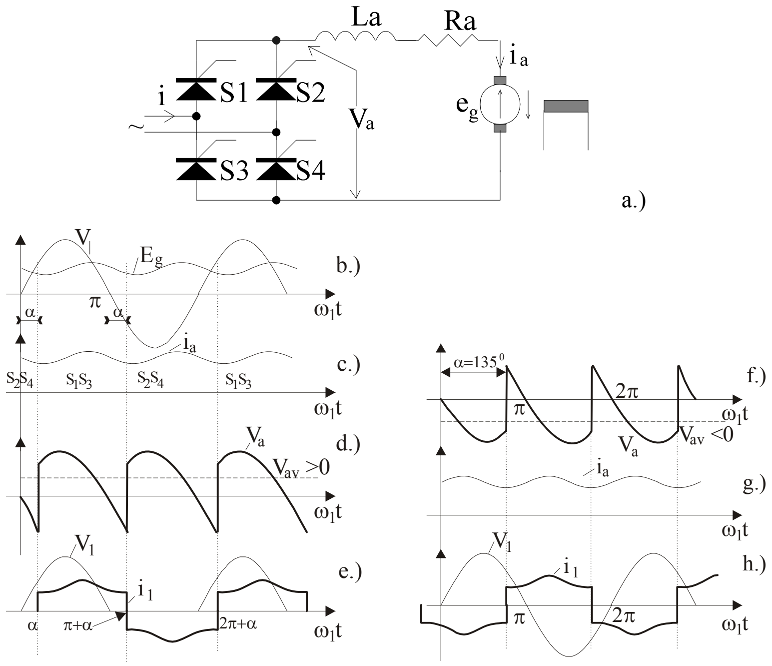


Figure 5.3. The DC brush motor fed through a single-phase full converter:

a.) the converter,

b.), c.), d.), e.) - voltage and current waveforms for motor action (α=45°),

f.), g.), h.) - for generator action (α = 135°).

a. Draw the voltage and current waveforms for steady state and finite motor armature inductance and α = 45° and α = 135°.

b. For a firing angle of α = 30° and rated motor current (rectifier regime), calculate: motor speed, torque and supply power factor, neglecting the motor current ripple.

c. By reversing the field current, the motor back e.m.f. E_g is reversed; for rated current, calculate: converter firing angle α' and the power fed back to the supply.

Solution:

a. The waveforms of currents and voltages are shown on Figure 5.3. It should be noted that because the motor inductance is not very high the armature current, and speed pulsate with time.

Motor action is obtained for α = 45° when both the motor and converter average input powers are positive. For generator action (α = 135°) the motor current remains positive but the motor average voltage V_a is negative. Thus both the motor and converter input average powers are negative for generating.

b. As the motor current is considered ripple-free, we use only the average values of voltage and current while the speed is also constant.

The average motor voltage V_{av} is

$$V_{av} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V\sqrt{2}\sin\omega_1 t \cdot d(\omega_1 t) = \frac{2\sqrt{2}}{\pi} V\cos\alpha =$$

$$= \frac{2\sqrt{2}}{\pi} 260\cos 30° = 202.44\ \text{V} \tag{5.31}$$

The motor torque T_e is

$$T_e = \frac{K_e\lambda_p}{2\pi} I_a = \frac{10}{2\pi} 40 = 63.6942\ \text{Nm} \tag{5.32}$$

The motor speed n is

$$n = \frac{V_{av} - R_a I_a}{K_e\phi} = \frac{202.44 - 0.2 \cdot 40}{10} = 19.444\ \text{rps} = 1166.64\ \text{rpm} \tag{5.33}$$

As the primary current is now rectangular (40A), the rms current is I_1 = 40A. The power from the supply, neglecting the losses, is $P_s = V_{av} \cdot I_a$ = $202.44 \cdot 40$ = 8097.6 W.

Thus, the supply power factor

$$PF = \frac{P_s}{V_1 \cdot I_1} = \frac{8097.6}{260 \cdot 40} = 0.7786 \tag{5.34}$$

c. For generator action the polarity of induced voltage should be reversed

$$e_g = -K_e\lambda_p n = -10 \cdot 19.44 = -194.44\ \text{V} \tag{5.35}$$

Thus the motor voltage V_a becomes

$$V_a = e_g + R_a I_a = -194.44 + 0.2 \cdot 40 = -186.44\ \text{V} \tag{5.36}$$

Finally,

$$\alpha = \cos^{-1}\left[\frac{V_a\pi}{2\sqrt{2}V}\right] = \cos^{-1}\left[\frac{-186.44 \cdot \pi}{2\sqrt{2} \cdot 260}\right] \approx 143° \tag{5.37}$$

The regenerated power P_{sg} is:

$$P_{sg} = V_a I_a = 186.44 \cdot 40 = 7457.6\ \text{W} \tag{5.38}$$

As for this regime either the flux λ_p *or the speed* n *changes sign, the motor torque opposes the motion providing generator braking.*

In the process, the motor speed and e.m.f. decrease and, to keep the current constant, the firing angle $\alpha > 90^0$ in the converter should be modified accordingly.

For the motor as above, having L_a = 2mH, calculate the following:

a. For α = 60° and constant speed n = 1200 rpm draw the voltage and current waveforms knowing that the motor current is discontinuous.

b. Calculate the motor current waveform for n = 600 rpm.

Solution:

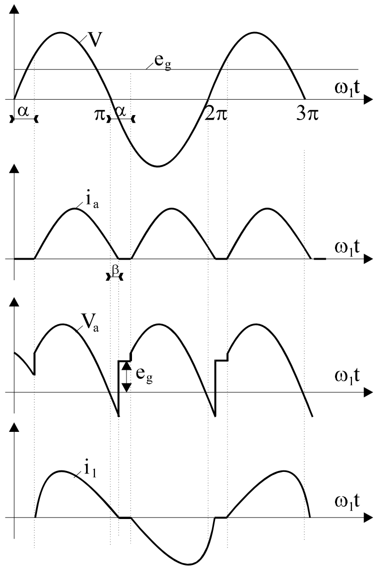


Figure 5.4. D.c brush motor supplied from a full converter – discontinuous current mode

a. As the motor electrical time constant $\tau_e = L_a / R_a = 2.10^{-3} / 0.2 = 10$ ms is small, when the voltage becomes zero (for $\omega_1 t = \pi$), the motor current decays quickly to zero along the angle β (from π to $\pi+\beta$) with $\beta<\alpha$ and thus the current is discontinuous. As α increases, for a given speed n (and e.m.f. e_g), the average voltage V_{av} increases with respect to the case of continuous current.

The discontinuity of current contributes to a sluggish dynamic response as, during zero current, motor internal torque control is lost. The situation occurs especially at low torques (and currents). Special measures such as flux weakening (reducing λ_p) for low torques leads to reduced α at high speeds, avoiding the discontinuity in current.

b. With the speed considered constant, the voltage equation is

$$V\sqrt{2}\sin\omega_1 t = R_a i_a + L_a \cdot \frac{di_a}{dt} + K_e \lambda_p n \; ; \qquad (5.39)$$

with $i_a = 0$ for $\omega_1 t_1 = \alpha$, the solution of above equation is

$$i_a = A \cdot e^{-tR_a/L_a} - \frac{K_e\lambda_p n}{R_a} + \frac{V\sqrt{2}\sin(\omega_1 t - \varphi_1)}{\sqrt{R_a^{\ 2} + \omega_1^{\ 2} L_a^{\ 2}}};$$

$$\varphi_1 = \tan^{-1}(\omega_1 L_a / R_a) \tag{5.40}$$

for $t = t_1 = \frac{\alpha_0}{\omega_1}; \quad (i_a)_{t_1} = 0$ and thus

$$\varphi_1 = \tan^{-1}\left(\frac{2\pi 60 \cdot 2 \cdot 10^{-3}}{0.2}\right) = 75.136° \tag{5.41}$$

$$i_a = 1402 \cdot e^{-100t} - 1000 + 470\sin(\omega_1 t - \varphi_1) \tag{5.42}$$

The current becomes zero again for $\omega_1 t_2 = \pi + \beta$

$$0 = 1402 \cdot e^{-\frac{100(\pi+\beta)}{2\pi 60}} - 1000 + 470\sin(\pi + \beta - 1.308) \tag{5.43}$$

The solution for β is $\beta \approx 12°$.

5.6. THE THREE-PHASE SEMICONVERTER

Consider a three-phase semiconverter (Figure 5.5) supplying a DC motor. The armature current is considered ripple free and the commutation is instantaneous.

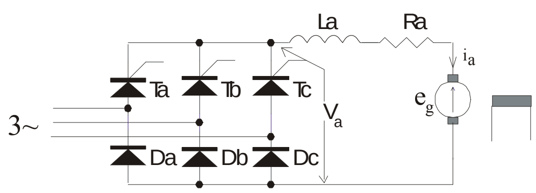
a. Draw the waveform of output voltage for $\alpha = 30°$.

b. Calculate the average (rms) voltage as a function of α.

c. For a line voltage (rms) $V_L = 220V$, $f = 60Hz$, $R_a = 0.2\Omega$, $K_e\lambda_p = 4Wb$ and an armature current $I_a = 50A$, determine the motor speed for $\alpha = 30°$.

Solution:

a. As seen from Figure 5.5b if the thyristor T_a is turned on with a delay angle $\alpha = 30°$ (point A) from the moment corresponding to $\alpha = 0$, it will conduct together with the diode D_b for 30° as long as V_{ab} is positive and higher than the other line voltages. At point B, T_a will conduct together with the diode D_c for 90° as $V_{ac}>V_{ab}$. At point C, after 120° of conduction, the thyristor T_a is turned off and T_b turned on and so on.



a.)

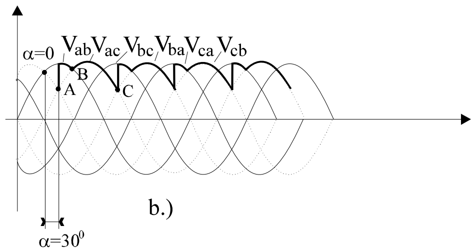


Figure 5.5. DC brush motor supplied through a three-phase semiconverter

b. The expression for average voltage is

$$V_{av} = \frac{3}{2\pi}\left[\int_{\alpha}^{\pi/3}\sqrt{2}V_L \sin(\omega_1 t + \pi/3)d(\omega_1 t) + \int_{\pi/3}^{\alpha+2\pi/3}\sqrt{2}V_L \sin(\omega_1 t)d(\omega_1 t)\right] =$$

$$= \frac{3\sqrt{2}V_L}{2\pi}(1+\cos\alpha) \tag{5.44}$$

For $\alpha > 60°$ the integration interval is from α to π with the same final result. The output average voltage V_{av} may not be negative and thus the converter may not be used as an inverter, confirming the single quadrant operation.

c. The motor voltage equation for ripple free current is

$$\frac{3\sqrt{2}V_L}{2\pi}(1+\cos\alpha) = R_a I_a + K_e \lambda_p n \tag{5.45}$$

$$\frac{3\cdot\sqrt{2}\cdot 220}{2\pi}\left(1+\cos\frac{\pi}{6}\right) = 0.2\cdot 50 + 4\cdot n \tag{5.46}$$

$$n = 66.66 \text{ rps} = 4000 \text{ rpm} \tag{5.47}$$

5.7. THE THREE-PHASE FULL CONVERTER – MOTOR SIDE

A three-phase full converter (Figure 5.6) supplying a DC motor has the data: line voltage $V_L = 220V$ (rms), $K_e\lambda_p = 10Wb$, $R_a = 0.2\Omega$, and neglect the ripples in the motor current.

a. For L_s equal to zero, calculate the output average voltage as a function of the delay angle α; and for $\alpha = 30°$, determine the motor speed for $I_a = 50A$.

b. Considering the AC source inductance $L_s \neq 0$, determine the expression of the output voltage as a function of α and I_a and calculate the motor speed for $I_a = 50A$, $\alpha = 30°$ and $L_s = 11mH$.

c. Calculate the overlapping angle at commutation for case b.

Solution:

a. The principle of operation is quite similar to that of the previous case. However, only a 60° conduction of each pair of thyristors occurs as shown in Figure 5.6.

In the absence of AC source inductances L_s ($L_s = 0$) the commutation is instantaneous. The average output voltage V_{av} is

$$V_{av} = \frac{3}{\pi}\int_{\alpha+\pi/3}^{\alpha+2\pi/3}\sqrt{2}V_L \sin\omega_1 t \cdot d(\omega_1 t) = \frac{3\sqrt{2}V_L}{\pi}\cos\alpha \quad (5.48)$$

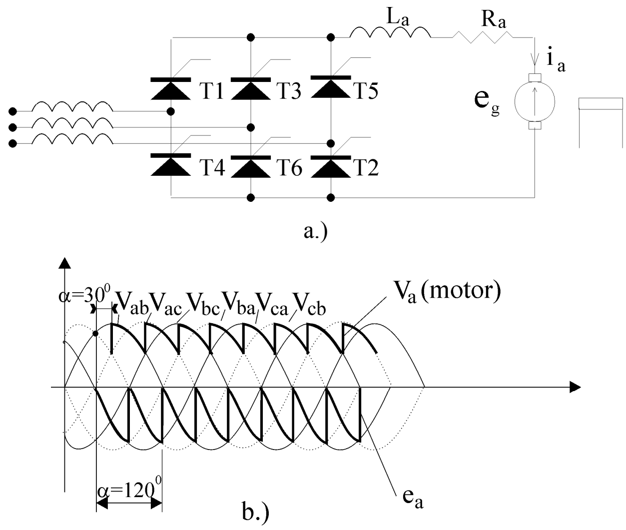


Figure 5.6. DC brush motor fed from a three-phase full converter

Consequently, $V_{av}<0$ ($V_{av}>0$) depending on α. For α<90 the average output voltage V_{av} is positive, that is rectifier mode; for α>90 $V_{av}<0$ and thus the inverter mode are obtained. The motor current remains positive irrespective of α. A two-quadrant operation is obtained.

The motor voltage equation is

$$\frac{3\sqrt{2}V_L}{\pi}\cos\alpha = R_a I_a + K_e\lambda_p n \quad (5.49)$$

$$n = \left(\frac{3\cdot 220\sqrt{2}\cos(\pi/6)}{\pi} - 0.2\cdot 50\right)/10 = 24.74\,\text{rps} = 1484.58\,\text{rpm} \quad (5.50)$$

b. The effect of AC source inductances L_s on current commutation is shown in Figure 5.7 for the case of constant motor current (similar to the diode full converter [chapter 3]). The commutation between T_1 and T_5 is not instantaneous anymore; an overlapping angle u occurs. The effect of actual commutation is, in fact, a reduction of output voltage V_d determined by the area A_u when a kind of short circuit occurs between phases a and c.

$$A_u = \int_{\alpha}^{\alpha+u} V_{Ls} d(\omega_1 t) = \int_{\alpha}^{\alpha+u} L_s \cdot \frac{di_a}{dt} d(\omega_1 t) = \omega L_s I_d \tag{5.51}$$

This is so since the current in phase a is zero for $\omega_1 t = \alpha$ and equal to I_d for $\omega_1 t = \alpha + u$. The average voltage is reduced by $3 / \pi\ A_u$.

Finally, the average voltage is equal to that already found for $L_s = 0$ minus $3 / \pi\ A_u$

$$V_d = \frac{3\sqrt{2} V_L}{\pi} \cos\alpha - \frac{3}{\pi} \omega_1 L_s I_d \tag{5.52}$$

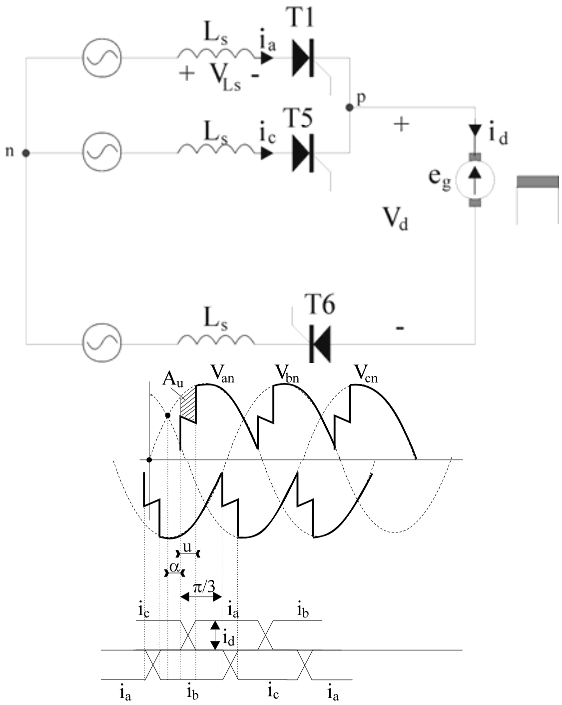


Figure 5.7. The current commutation in the presence of AC source side inductances L_s.

As we can see, the angle u is not required to calculate the output voltage V_d. However, it is needed to ensure reliable operation in the inverter mode ($\alpha > 90°$). The second Kirchoff law for phase a and c during commutation provides the equation

$$V_{an} - L_s \cdot \frac{di_a}{dt} = V_{cn} - L_s \cdot \frac{di_c}{dt} \tag{5.53}$$

with $i_c + i_a = i_d$, that is $\frac{di_a}{dt} = -\frac{di_c}{dt}$ (5.54)

Thus,

$$V_{an} - V_{cn} = V_{ac} = 2L_s \cdot \frac{di_a}{dt} \quad (5.55)$$

with $V_{ac} = \sqrt{2}V_L \sin\omega_1 t$ (5.56)

$$\int_{\alpha}^{\alpha+u} V_{ac} d(\omega_1 t) = 2\omega_1 L_s I_d \quad (5.57)$$

and finally,

$$\cos(\alpha + u) = \cos\alpha - \frac{2\omega_1 L_s I_d}{\sqrt{2}V_L} \quad (5.58)$$

Thus knowing α and I_d we may calculate the overlapping angle u.

In the inverter mode (α>90°) the commutation should be finished before α + u = π in order to allow the turn-off time t_{off} required for the recombination of charged particles in the thyristors ($\frac{\pi - (\alpha + u)}{\omega_1} > t_{off}$) for negative voltage along the turning-off thyristors.

The rectifier voltage V_d for α=π / 6 and I_d = 50 is

$$V_d = \frac{3\sqrt{2} \cdot 220}{\pi}\cos\frac{\pi}{6} - \frac{3}{\pi}2 \cdot \pi \cdot 60 \cdot 50 \cdot 10^{-3} = 239.43 \text{ V} \quad (5.59)$$

$$V_d = R_a I_a + K_e \lambda_p n$$

$$239.43 = 0.2 \cdot 50 + 10 \cdot n; \quad n = 22.943 \text{ rps} = 1376.58 \text{ rpm} \quad (5.60)$$

The reduction of output voltage for the same α due to commutation with 18V has contributed to a notable reduction in speed, for 50A, from 1484.58 rpm to 1376.58 rpm.

c. To calculate the overlapping angle u we use the expression derived above.

$$\cos(\alpha + u) = \cos\alpha - \frac{3\omega_1 L_s I_d}{\sqrt{2}V_L} = \cos\frac{\pi}{6} - \frac{3 \cdot 2\pi \cdot 60 \cdot 50 \cdot 10^{-3}}{\sqrt{2} \cdot 220}$$

$$30° + u = 46.78°; \quad u = 16.78° \quad (5.61)$$

A considerable value for u has been obtained.

5.8. THE THREE-PHASE FULL CONVERTER – SOURCE-SIDE ASPECTS

For the three-phase full converter and DC motor in the previous example with $L_s = 0$ for $\alpha = 0°$, $\alpha = 45°$ with $I_d = 50A$, determine:

a. The waveforms of AC source current and its harmonics factor.

b. The rms of fundamental source current and of the total source current.

c. The displacement power factor for $L_s = 0$ and $L_s = 1mH$ for $\alpha = 45°$.

d. Calculate the line voltage and voltage distortion due to $L_s = 1mH$ and $\alpha = 45°$.

<u>Solution</u>:

a. As the motor armature current is considered constant, the AC source current is rectangular as shown in Figure 5.8a and b.

The presence of AC source inductances $L_s \neq 0$ leads to the overlapping angle $u \neq 0$.

b. The rms value of the current fundamental, I_{a1}, is

$$I_{a1} = \frac{2\sqrt{3}}{\pi} \cdot \frac{I_d}{\sqrt{2}} = \frac{\sqrt{6}}{\pi} \cdot I_d = 0.78 \cdot 50 = 39 \text{ A} \tag{5.62}$$

For the total AC source current which is made of rectangular 120^0 wide "blocks",

$$I_a = \sqrt{\frac{2}{3}} \cdot I_d = 0.816 \cdot 50 = 40.8 \text{ A} \tag{5.63}$$

In the absence of L_s, the displacement power factor (DPF) angle is equal to α and thus the DPF is

$$DPF = \cos\varphi_1 = \cos\alpha = \begin{cases} 1 \text{ for } \alpha = 0 \\ 0.709 \text{ for } \alpha = 45° \end{cases} \tag{5.64}$$

In the presence of L_s, the displacement power factor is approximately

$$DPF = \cos\left(\alpha + \frac{u}{2}\right) \tag{5.65}$$

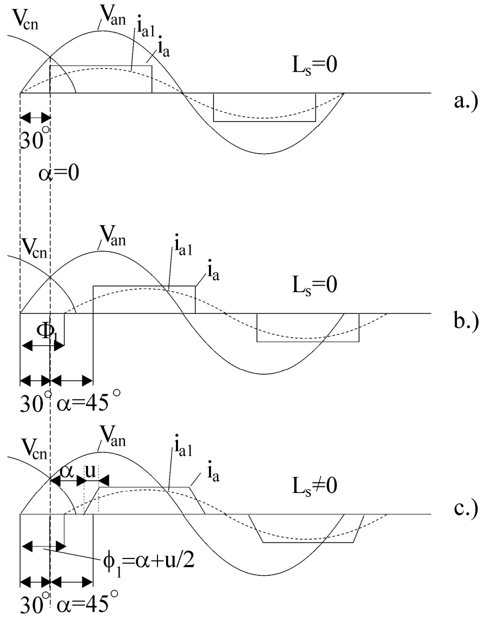


Figure 5.8. AC source currents in a three-phase full converter and constant motor current: a.) for $\alpha = 0$, $L_s = 0$; b.) for $\alpha = 45°$, $L_s = 0$; c.) for $\alpha = 45°$, $L_s = 1$mH

To calculate u for $\alpha = 45°$, we use (5.58)

$$\cos(\alpha + u) = \cos\alpha - \frac{3\omega_1 L_s I_d}{\sqrt{2}V_L} = 0.707 - \frac{3\cdot 2\pi\cdot 60\cdot 50\cdot 10^{-3}}{\sqrt{2}\cdot 220} = 0.5248$$

$$45 + u = 58.34°; \quad u = 13.34° \tag{5.66}$$

Finally,

$$DPF = \cos\left(45 + \frac{13.34}{2}\right) = 0.62$$

The current commutation in the presence of L_s produces a further reduction of the displacement power factor.

In general, the DPF decreases with α increasing, which constitutes a notable disadvantage of phase delay AC-DC converters. Special measures are required to improve DPF with α increasing.

d. The AC source current overlapping during commutation produces notches in the line voltage. From Figure 5.9. we may obtain $V_{ab} = V_{an} - V_{bn}$ of the waveform shown in Figure 5.9.

Considering u as small, the deep notch depth is considered equal to $\sqrt{2}V_L \sin\alpha$ and thus the notch width u is approximately:

$$u = \frac{A_u}{\text{notch depth}} = \frac{2\omega_1 L_s I_d}{\sqrt{2}V_L \sin\alpha} \tag{5.67}$$

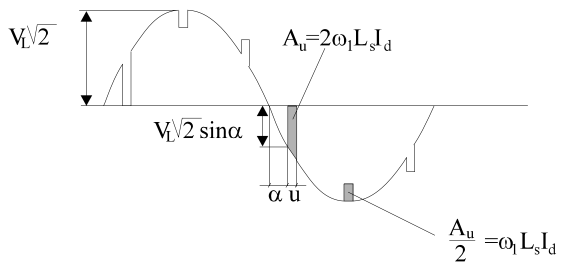


Figure 5.9. AC source-line voltage notches due to Ls (commutation)

The depth of shallow notches is considered half of that of deep notches. IEEE standard 519-1981 suggests the limitation of line notches to 250 μs (5.4. electrical degrees) and the deep notch depth to 70% of rated peak line voltage in order to perform satisfactorily.

Special filtering is required to cope with more recent standards. The voltage distortion due to notches depends on the harmonics currents I_ν and the AC source inductance L_s

$$\text{voltage\%THD} = \frac{\sqrt{\left[\sum_{\nu>1}\left(I_\nu \cdot \nu\omega_1 L_s\right)^2\right]}}{V_{\text{phase (fundamental)}}} \cdot 100 \tag{5.68}$$

with $I_\nu \approx \frac{I_{a1}}{\nu} = \frac{\sqrt{6}}{\pi} \cdot \frac{I_d}{\nu}$ due to the almost rectangular form of AC source current.

5.9. THE DUAL CONVERTER - FOUR-QUADRANT OPERATION

A high power DC motor drive sometimes has to undergo four-quadrant operation. Two full converters are connected back to back for this purpose (Figure 5.10).

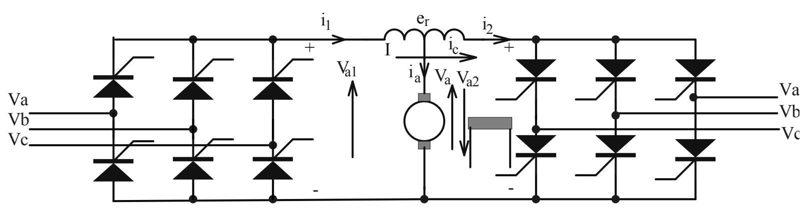


Figure 5.10. Dual converter with circulating current supplying a DC brush motor

a. Assuming that the converters are ideal and produce pure DC output voltages with one converter as rectifier and the other as inverter, calculate the relationship between the delay angles α_1 and α_2 in the two converters.

b. With $\alpha_1 + \alpha_2$ as above, calculate the circulating current between the two converters and show the voltage and current actual output waveforms. The numerical data are $V_L = 220V$, $\omega_1 = 377.8rad/s$, $L = 10mH$, $\alpha_1 = 60°$.

Solution:

a. In an ideal dual converter the voltages produced by the two full converters should be equal and opposite.

By now we know that

$$V_{a1} = V_{max} \cdot \cos\alpha_1$$
$$V_{a2} = V_{max} \cdot \cos\alpha_2 \tag{5.69}$$

with $V_a = V_{a1} = -V_{a2}$ it follows that $\cos\alpha_1 + \cos\alpha_2 = 0$.

Hence, $\alpha_1 + \alpha_2 = 180°$ (Figure 5.11). In the ideal converter the load voltage is equal to the converter output voltages and thus the current may flow equally through either converter.

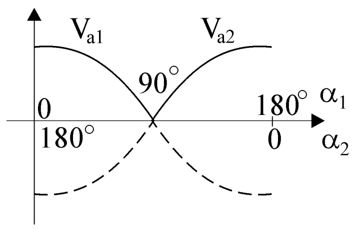


Figure 5.11. Ideal dual converter voltages

b. A real (nonideal) converter produces a voltage with ripples. The ripple voltages of the two converters are out of phase (Figure 5.12). The instantaneous voltage difference produces a circulating current which is limited through a reactor L.

With

$$V_{a,b,c} = \frac{V_L\sqrt{2}}{\sqrt{3}}\sin\left[\omega_1 t - (i-1)\frac{2\pi}{3}\right] \tag{5.70}$$

during the interval

$$\frac{\pi}{6} + \alpha_1 < \omega_1 t < \frac{\pi}{6} + \alpha_1 + \frac{\pi}{3} \tag{5.71}$$

$$V_{a1} = V_a - V_b$$
$$V_{a2} = -(V_c - V_b)$$
$$e_r = V_{a1} - V_{a2} = V_a + V_c - 2V_b = -3V_b \tag{5.72}$$

The circulating current i_c is

$$i_c = \frac{1}{\omega_1 L}\int_{\alpha_1+\frac{\pi}{6}}^{\omega_1 t} e_r dt = \frac{\sqrt{6}V_L}{\omega_1 L}\left[\cos\left(\omega_1 t - \frac{2\pi}{3}\right) - \cos\left(\alpha_1 - \frac{\pi}{2}\right)\right] \quad (5.73)$$

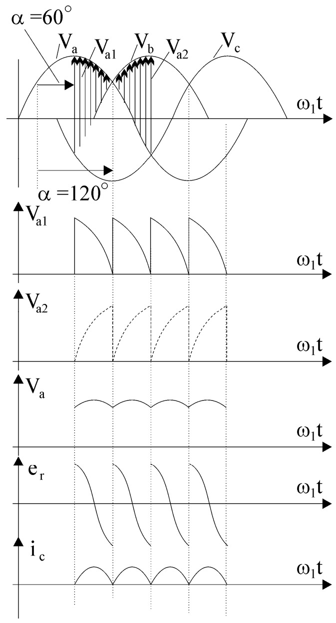


Figure 5.12. The real dual converter

When the motor current is zero, the converter currents are equal to the circulating current $i_1 = i_2 = i_c$ ($i_a = 0$). Consequently, the converters have a continuous current though the load current is zero. However for $\alpha_1 = 60°$ and $\alpha_2 = 120°$ the peak value of circulating current occurs at $\omega_1 t = 2\pi/3$

$$(i_c)_{peak} = \frac{V_L\sqrt{6}}{\omega_1 L}\left[1-\cos\frac{\pi}{6}\right] = \frac{220\sqrt{6}}{2\cdot\pi\cdot 60\cdot 10^{-2}}[1-0.867] = 19.02 \text{ A} \quad (5.74)$$

If the load current i_a is constant (no ripples), the first converter ($\alpha_1 = 60°$) carries $i_a + i_c$ while the second converter ($\alpha_1 = 120°$) has the circulating current i_c only. Thus the first converter is “overloaded” with the circulating current. However, for low load current, the discontinuous current mode in the converters is avoided as shown above. This could be an important advantage in terms of control performance.

5.10. AC BRUSH SERIES (UNIVERSAL) MOTOR CONTROL

The universal motor is AC voltage supplied but it is still a brush (mechanical comutater) series connected motor (Chapter 3), Figure 5.13.

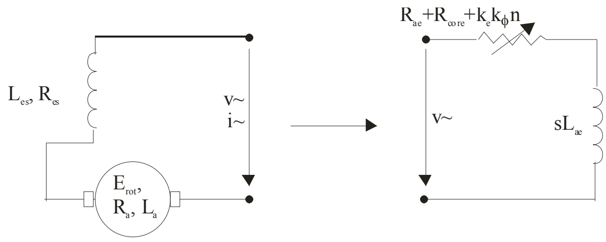


Figure 5.13. The equivalent scheme of universal motor

The motor voltage equation is (Chapter 3)

$$L_{ae}\frac{di}{dt} \approx v - \left(R_{ae} + R_{core} + k_e k_\Phi n\right)i; \quad T_e = \frac{k_e k_\Phi i^2}{2\pi} \tag{5.75}$$

The mechanical equation is straightforward

$$J2\pi\frac{dn}{dt} \approx \frac{k_e k_\Phi i^2}{2\pi} - Bn - T_{load} \tag{5.76}$$

All core losses are lumped into the stator while in the motion equation they are left out for simplicity. Magnetic saturation of the magnetic circuit is considered constant or absent (L_{ae} = ct, k_Φ = const.).

The investigation of transients with any input voltage waveform and load torque perturbation is rather straightforward for such a second order system.

The motion induced voltage (e.m.f.), E_{rot}, may be considered as an additional, variable resistance voltage drop ($k_e k_\Phi ni$), eqn. (5.75).

The torque is essentially proportional to current squared and, for given voltage, it is proportional to voltage squared.

An AC voltage changer is required.

At start (n=0) the machine is represented by a small resistance R_{ae}+R_{core} plus the inductance L_{ae}. So the electric circuit of the machine at start is strongly inductive.

On the other end, at high speed, due to the large e.m.f. (E_{rot}) − eqn. (5.75) − the machine equivalent circuit is notably resistive.

The equivalent circuit is composed simply of an equivalent inductance L_{ae} − relatively constant − and a resistance R_{en} = (R_{ae}+R_{core}+$k_e k_\Phi n$) which strongly increase with speed (Fig. 5.13).

A typical AC voltage charger − which may be used also as a power switch − may be obtained by using two antiparalleled thyristors which may be assembled into a single bidirectional power switch (Fig. 5.14) − the Triac.

The Triac is turned on by applying a short resistive current pulse on the thyristor gate. The thyristor will turn off when the current decays naturally to zero. Then the thyristor for the negative voltage polarity the antiparalleled thyristor is turned on. As the turn-on angle α − with respect to zero crossing of the voltage waveform – increases, the average voltage decreses. But the average voltage depends also on the machine equivalent resistance R_e which increases with speed.

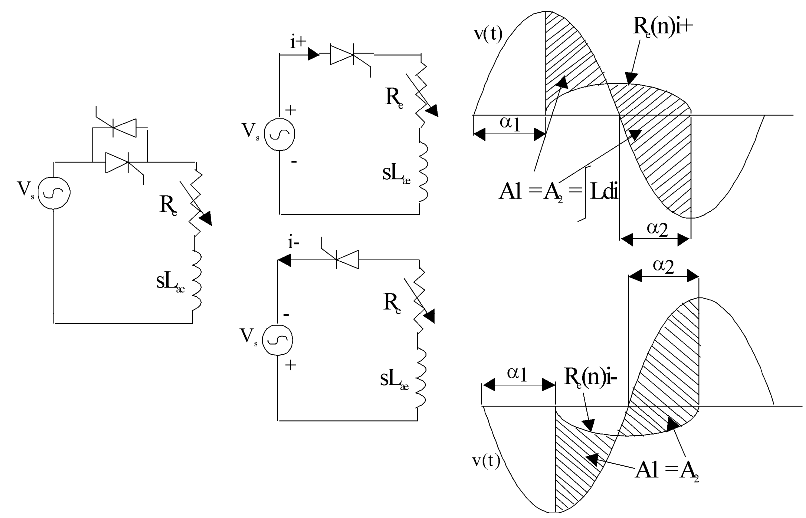


Figure 5.14. The Triac variac and the current

Let us consider separately the two voltage polarities (Figure 5.14).

The difference between supply voltage v(t) and the resistance voltage $R_e(n)i$ is equal to the voltage drop along the machine inductance L_{ae}:

$$v_{Lae}(t) = L_{ae}\frac{di}{dt} = v_s(t) - R_e(n)i \tag{5.77}$$

$$\int_{\alpha 1}^{\pi+\alpha 2} v_{Lae}(t)dt = 0 \tag{5.78}$$

The current goes to zero when areas A1 and A2 on Figure 5.14 become equal to each other.

It is evident that for the universal motor the current goes to zero after the voltage changes polarity by the angle α_2. However this delay angle decreases with speed as the circuit becomes more and more resistive as the e.m.f. increases.

There may be situations with $\alpha_2 > \alpha_1$ and $\alpha_2 < \alpha_1$. In the first case the current will be continuous while in the second case it will be discontinuous.

In general α_1 has to be larger than the displacement power factor angle φ1 of the equivalent circuit, to secure the fundamental output voltage control by the Triac.

So, at low speed α_1 should be large to reduce the voltage fundamental and will decrease with speed. But the output voltage is full of harmonics as only parts of the sinusoid are active. Also there are harmonics in the machine (input) current (Fig. 5.15). An input power filter is required.

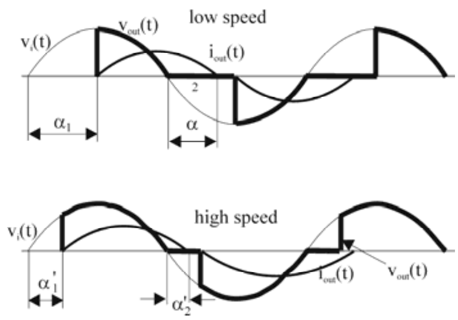


Figure 5.15. Output voltage and current waveforms at low and high speed

For constant speed and given delay angle α_1, eqn (5.77) has an analytical solution:

$$i = Ae^{-\frac{Re(n)}{Lae}t} + B\sin(\omega_1 t - \varphi_1);\;\; B = \frac{V\sqrt{2}}{\sqrt{R_e^2(n) + \omega_1^2 L_{ae}^2}};\;\; \varphi_1 = \tan^{-1}\frac{\omega_1 L_{ae}}{R_e(n)} \quad (5.79)$$

with $i = 0$ at $\omega_1 t = \alpha_1$.

Consequently:

$$A = -B\sin(\alpha_1 - \varphi_1)e^{\frac{R_e(n)\alpha_1}{L_{ae}\omega_1}} < 0 \quad (5.80)$$

So, $\alpha_1 > \varphi_1$ to secure predominantly positive sinusoidal current for positive voltage polarity.

The angle α_2 within negative polarity of voltage where the current decays to zero is obtained by applying (5.78). The obtained nonlinear equation may be solved numerically for α_2.

It is also evident that the output voltage fundamental V_{1out} is nonlinearly dependent on $(\pi/2-\alpha_1)$ or on $(1-\sin\alpha_1)$.

A robust speed controller is required to overcome this difficulty.

A generic control system for the universal motor is presented in Fig. 5.16.

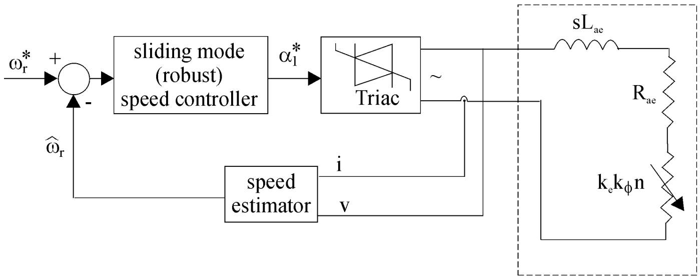


Figure 5.16. Generic control system for universal motors

A speed estimator is required for close loop control. In less demanding applications a well commissioned feedforward open loop control $\alpha_1^*(\omega_r^*)$ with given limited speed ramps may be used to avoid the speed estimator and the speed loop control.

Note: The low cost of universal motor drive at low power levels has secured its presence today in some home appliances and hand-held tools (with natural limited operation life) despite the occurrence of brushless drives. For more details see Reference 6.

5.11. SUMMARY

- Controlled rectifiers, also known as phase delay rectifiers, of various basic configurations, with zero and nonzero source inductances, have been presented in interaction with DC brush motors at constant speed.
- Single-phase and three-phase full converters provide for bidirectional power flow for positive output current and positive and negative average output voltages. Consequently, for positive speeds regenerative braking is possible only if the field current direction is changed as negative e.m.f. is required.
- Dual converters with circulating currents, requiring 12 PESs in three phase configurations provide for four-quadrant operation while avoiding the discontinuous current mode at the price of overloading one of the two converters.
- The source inductance produces an overlapping during phase commutation which results in a kind of resistive-like output voltage drop in the rectifier.
- The power factor decreases with a decrease in DC output voltage in phase delay rectifiers. Forced commutation or special complex configurations (for high powers) may solve this problem [3-6].
- All controlled rectifiers produce current harmonics and voltage notches on the AC source side and special input filters are required to reduce them to acceptable standardized levels.

5.12. PROBLEMS

5.1. A three-phase rectifier with controlled null (Figure 5.17) supplies a load made of a resistance R_s and an inductance L_s. The AC supply phase voltage is 120V (rms); $R_s = 10\Omega$; and the transformer ratio $K = w_1/w_2=2$. For $L_s = 0$ and $L_s = \infty$, determine:

5.2. The output and transformer secondary voltages and current waveforms for the delay angle $\alpha_1 = \pi / 3$.

5.3. The average values of output voltage and current.

5.4. The waveform of transformer primary current.

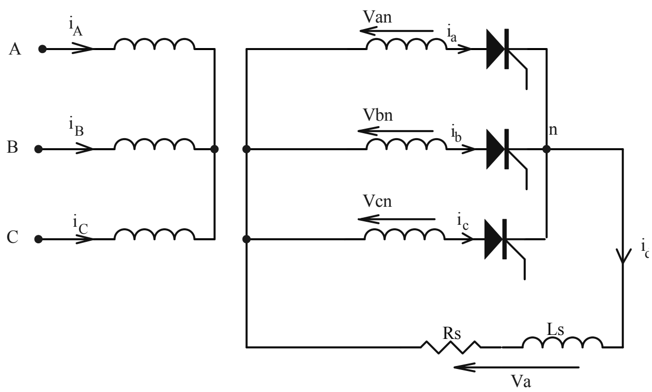


Figure 5.17. Three-phase rectifier with controlled null

5.5. A DC series motor is supplied through a single phase semiconverter with a free-wheeling diode (Figure 5.18). Depict the waveforms of the induced voltage e_g and motor voltage V_a for discontinuous and continuous current.

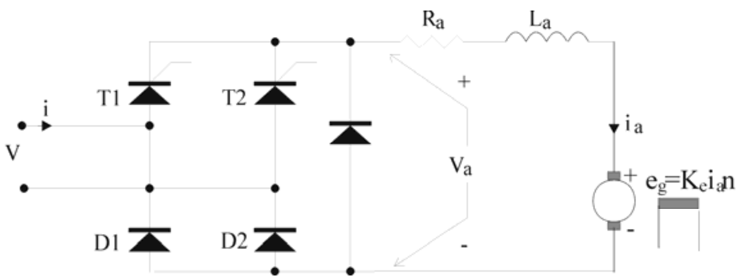


Figure 5.18. DC series motor supplied through a single-phase semiconverter

5.6. For problem 5.2 find analytical expressions of armature current for discontinuous and continuous modes for constant speed, with saturation neglected. Find the condition to determine the minimum value of L_a for which the current is still continuous. Determine an expression of average output voltage for the continuous current.

5.7. A single-phase full converter [7] uses power transistors and is controlled through pulse width modulation with a sinusoidal carrier with n pulses per semiperiod (Figure 5.19). Obtain the expressions of average output voltage, current harmonics, power factor (PF), displacement factor (DF) and current harmonics factor (HF).

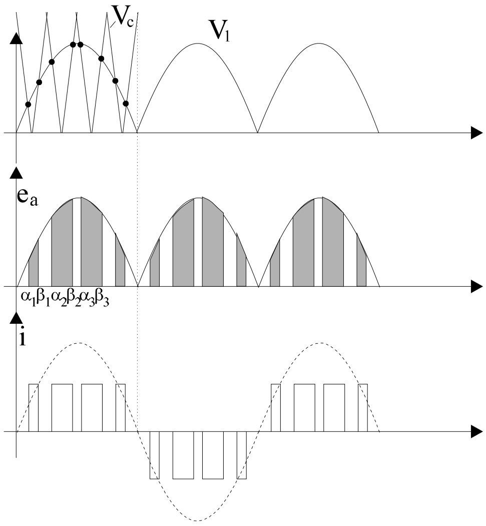


Figure 5.19. Single-phase full converter with PWM control

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