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Chapter 4

DC BRUSH MOTORS FOR DRIVES

DC brush or DC commutator machines have traditionally been used in variable speed drives in the range of low (a few watts) to medium, 10MW, power ratings at low speeds. The popularity of DC brush motors in variable speed drives is primarily due to the lower cost of a single-stage (rectifier) power electronic converter (PEC) required for two-quadrant operation (one direction of motion). Four-quadrant operation, however, implies rather involved PEC configurations and controls.

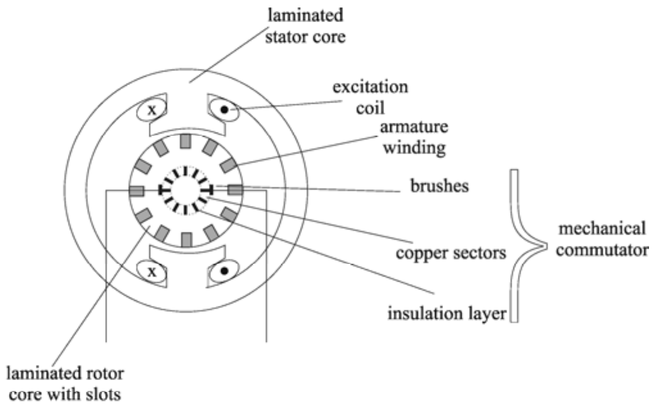
In this chapter we will discuss DC brush motor basic topologies, state space equations, steady-state curves, losses, and transfer functions, as they will be used in subsequent chapters on DC brush motor drives.

4.1 BASIC TOPOLOGIES

As in any electric motor, the DC brush motor has two main parts: the stator (fixed) part and the rotor (movable) part. The rotor may be cylindrical (Figure 4.1a) or disk-shaped and contains a symmetric winding made of identical coils connected in series to the insulated copper sectors of the so-called mechanical commutator. The unipolar current injected through the brushes is converted into bipolar current in the rotor coils through the commutator copper sectors, in pace with the rotor position. The mechanical commutator is, in fact, an inverter (DC-AC converter) which changes the frequency from zero to $f_n = p.n$ where 2p is the number of stator and rotor poles (semi-periods) and n is the rotor speed.

Electromagnetic DC excitation (EE) may be replaced by permanent magnets (PMs) – Figure 4.1b and 4.2. High energy PMs may be replaced by a constant excitation (field) current fictitious (or superconducting) coil. The PM stator is ideally lossless as the PM magnetization (or demagnetization) losses are fairly small.

The disk rotor does not have an iron core and thus the rotor winding inductance is fairly small. Moreover, the design current density is higher than for coils in iron core slots, due to direct air exposure. Consequently, the electrical time constant of the disk DC brush motor is the lowest known (approximately 1ms for 1kW, 3000 rpm motors).



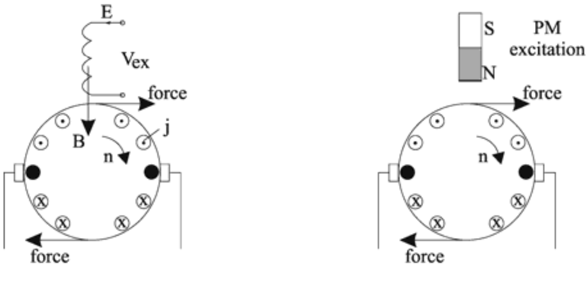


Figure 4.1. Cylindrical DC brush motor a.) topology, b.) schematics with electromagnetic excitation, c.) schematics with permanent magnets (PMs).

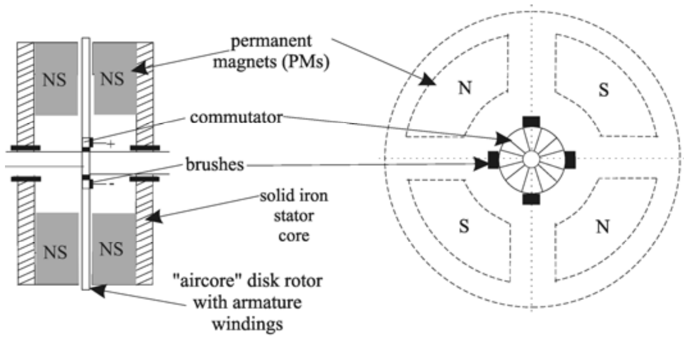


Figure 4.2. Disk DC brush motor (2p = 4poles)

a.) cross section; b.) axial view

Unfortunately, the power per unit mass of a disk DC brush motor is limited by the mechanical fragility of the “aircore” rotor, to 2-3kW and less than 6000 rpm. On the other hand, the cylindrical DC brush motor is power limited by the commutator to 10MW at low speeds for rotors with coils in slots of laminated iron cores. “Air core” cylindrical rotor windings used for small powers are now proposed to improve commutation (due to inductance reduction) and thus increase the power above 10 MW at very low speeds.

For details on DC brush motors, refer to [1].

4.2. THE MOTION-INDUCED VOLTAGE (e.m.f.)

As the rotor conductors (Figure 4.1) move in the field produced by the field current (or the PM), an e.m.f. is induced in each rotor coil. The rotor coils are connected in series between neighboring brushes through the copper sectors of the mechanical commutator.

As seen from Figure 4.1, there are at least two current paths in parallel. In general, there are 2a paths. The number of current paths depends on the number of poles, 2p, and the type of the armature winding (lap or wave type). The total number of conductors per rotor periphery is N and the flux linkage per pole is (half-period of excitation field in the airgap) λ_p.

The motion-induced voltage in the rotor, measured at the brushes, E, is also proportional to rotor speed n.

Finally:

$$E = K_e \cdot n \cdot \lambda_p; \quad K_e = \frac{p}{a} \cdot N \tag{4.1}$$

The pole flux linkage λ_p is proportional to the average flux density in the airgap per pole B_{gav}, to the pole pitch τ and to the core stack length L

$$\lambda_p = B_{gav} \cdot \tau \cdot L \tag{4.2}$$

As seen from Figure 4.1 the motion induced voltage (e.m.f.) in each coil does not change sign under one pole as the fixed brushes collect the voltage of the coils temporarily located (though in continuous motion) under one pole (field) polarity. The armature current changes polarity when the respective coil is short-circuited by the brushes.

This phenomenon is called mechanical commutation of currents. For more details on the DC brush motor topology, principle, performance and design, see [1].

4.3. PERFORMANCE EQUATIONS: d-q MODEL

The DC brush motor schematics in Figure 4.1 has ideally an electrical 90° spatial phase difference between the stator (excitation, or PM) magnetic field and rotor (armature) magnetic field (brush axis). Consequently, in the absence of magnetic saturation, there is no interaction (transformer induced voltage) between excitation and armature windings. The excitation (PM) circuit may be termed the field circuit while the rotor (armature) winding may be termed the torque circuit. *Thus the DC brush motor allows for separate (decoupled) control of field and torque currents (or torque)*, which is an extraordinary built-in property of DC brush motor. Though the stator excitation winding may also be series-connected to the brushes, separate excitation is considered in what follows.

There is an interaction between the stator and rotor windings only through motion voltage, E, induced in the rotor by the stator excitation current. The state-space equations of a DC brush motor are

$$V_{ex} = R_e \cdot i_e + L_e \cdot \frac{di_e}{dt}$$
$$V_a = R_a \cdot i_a + E + L_a \cdot \frac{di_a}{dt} \qquad (4.3)$$

The pole flux, λ_p, in the rotor windings, is dependent of i_e, in the absence of magnetic saturation, and also of i_a at high rotor currents if magnetic saturation occurs.

So, in general

$$\lambda_p = G \cdot i_e \qquad (4.4)$$

where G is the "motion-produced" inductance between the stator and rotor orthogonal windings. The electromagnetic torque, T_e, expression is obtained from the electromagnetic power P_e

$$T_e = \frac{P_e}{2\pi n}; \quad P_e = E \cdot i_a \qquad (4.5)$$

Thus

$$T_e = \frac{E \cdot i_a}{2\pi n} = \frac{K_e}{2\pi} \cdot \lambda_p \cdot i_a \qquad (4.6)$$

The DC brush motor parameters are R_e, R_a, as resistances and L_e, G, L_a as inductances. We should add the inertia, J, as defined through motion equation

$$J 2\pi \cdot \frac{dn}{dt} = T_e - T_{load} - B \cdot n; \quad \frac{d\theta_r}{dt} = 2\pi n \qquad (4.7)$$

where T_{load} is the load torque and B is the friction torque coefficient.

4.4. STEADY-STATE MOTOR CHARACTERISTICS

Steady-state means constant speed (dn/dt = 0) and constant currents (i_e = ct, i_a = ct). The steady-state voltage equations, obtained from (4.1)-(4.3) are

$$V_{ex} = R_e \cdot i_e \qquad (4.8)$$

$$V_a = R_a \cdot i_a + K_e \cdot n \cdot \lambda_p \tag{4.9}$$

$$T_e = \frac{K_e}{2\pi} \cdot \lambda_p \cdot i_a = T_{load} + B \cdot n \tag{4.10}$$

The main characteristic of a motor is the torque (T_e) versus speed (n, or Ω_r) curve called the mechanical characteristics.

From (4.8)-(4.10)

$$V_a = K_e \cdot n \cdot \lambda_p + R_a \cdot \frac{T_e \cdot 2\pi}{K_e \cdot \lambda_p} \tag{4.11}$$

Modifying the speed, for a given electromagnetic torque (T_e) may be done through:

- voltage, V_a, control;
- flux, λ_p, control.

As apparent from (4.11), it is also possible to add a resistance in series with R_a, to modify speed. This is, however, an energy consuming method which is to be avoided in modern electric drives.

The torque/speed curves obtained through voltage and flux control, all straight lines, are shown on Figure 4.3 in relative units.

In Figure 4.3, n_b, T_{eb} and λ_{pb} are base-speed, base-torque and base-flux values (continuous duty, rated (maximum) voltage).

Above the base-speed flux-weakening at constant armature voltage V_a (limited by the PEC capabilities) is performed, in general, at constant power P_e up to n_{max} / n_b = 2-3. Obviously, in PM brush motors flux weakening is not possible.

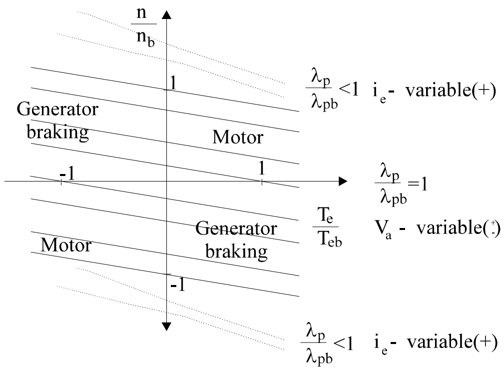


Figure 4.3. The torque/speed curves for variable speed

Also it is evident that both speed control methods illustrated in Figure 4.3, are of the high energy conversion ratio type. So far we have not discussed motor losses though they are crucial in defining the capability of an electric drive.

4.5. DC BRUSH MOTOR LOSSES

The performance equations already show two kinds of losses in the motor: the armature (copper) losses represented by R_a and the friction (mechanical) losses P_{mec}

$$P_{mec} \approx B \cdot \omega_r \tag{4.12}$$

In reality, the friction (mechanical) losses are a complex function of speed, depending on application. For example, in an electric train, the wheel-track and head and lateral surface air-drag losses all constitute load at constant speed and have complex mathematical expressions.

The N-S-N-S pole sequence in the stator poles field produces hysteresis and eddy current losses in the rotor laminated iron core. They are called core losses P_{iron}

$$P_{iron} \approx \left[C_h (p\,n) + C_e (p\,n)^2\right] \cdot B_{iron}^{\ 2} \cdot G_c \tag{4.13}$$

The first term represents the hysteresis losses while the second takes care of the eddy current losses. B_{iron} is the flux density in the rotor core and Gc the respective core weight.

The slotted rotor core has two main zones: the teeth and back core (or yoke). Further, the slot opening presence leads to stator-pole shoe core loss due to rotor (armature) winding m.m.f. These are called additional (surface) core losses, P_{add}.

Now the armature resistance R_a includes, in principle, the brush, brush-commutator contact and the equivalent resistance of commutator sectors. As the brush-commutator contact surface electrical resistance depends on many factors such as speed n, current i_a, brush-spring tension, especially in low voltage motors, the commutator losses, P_{com}, are to be calculated separately

$$P_{com} = \Delta V_b (i_a, n) \cdot i_a \tag{4.14}$$

Finally, the excitation winding losses, P_{ex}, have to be considered

$$P_{ex} = R_e \cdot i_e^{\ 2} \tag{4.15}$$

Figure 4.4 summarizes the breakdown of motor losses as developed above.

The efficiency η is:
$$\eta = P_{output} / P_{input} \tag{4.16}$$

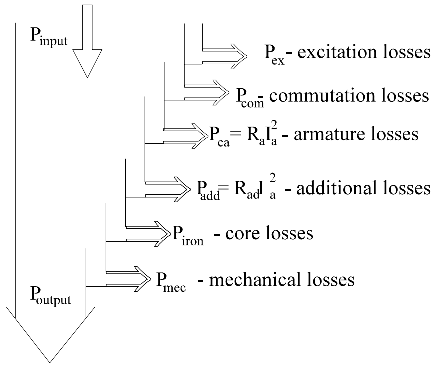


Figure 4.4. Loss breakdown in DC brush motors

Example 4.1. Steady state performance

A DC motor with separate excitation has the following data: rated power P_n = 3kW, rated (base) speed n = 1200 rpm, $P_{com} = 0.5\%P_n$, $P_{add} = 0.5\%P_n$, $P_{iron} = P_{mec} = 1\%P_n$ and $P_{ca} = 4\%P_n$, at rated (maximum) voltage V_{an} = 110V. The excitation losses are neglected.

Calculate:

a. total losses and rated efficiency, η_n;

b. rated current, I_n, and armature resistance, R_a;

c. brush voltage drop, ΔV_b;

d. motion-induced voltage, E_n;

e. rated (base) electromagnetic torque, T_{eb};

f. shaft torque, T_{load};

g. ideal no-load speed, n_0;

h. the armature voltage, V_{ag}, required to produce rated generator braking torque at rated speed.

Solution:

a. The total losses $\sum P$ are

$$\sum P = P_{co} + P_{com} + P_{add} + P_{mec} + P_{iron} = \left(4.0 + 0.5 + 0.5 + 1 + 1\right) \cdot \frac{3000}{100} = 210 \text{ W} \tag{4.17}$$

The efficiency:

$$\eta_n = \frac{P_n}{P_n + \sum P} = \frac{3000}{3000 + 210} = 0.9345 \tag{4.18}$$

b. The rated current I_n is

$$I_n == \frac{P_n}{\eta_n \cdot V_n} = \frac{3000}{0.9345 \cdot 110} = 29.18 \text{ A} \quad (4.19)$$

The armature resistance R_a is

$$R_a = \frac{P_{ca}}{I_n^2} = \frac{0.04 \cdot 3000}{29.18^2} = 0.140 \, \Omega \quad (4.20)$$

c, d. From the voltage equation (4.9), adding the brush voltage drop ΔV_b, we obtain

$$E_n = V_n - R_a \cdot I_n - \Delta V_b \quad (4.21)$$

with

$$\Delta V_b = \frac{P_{com}}{I_n} = \frac{0.005 \cdot 3000}{29.18} = 0.514 \text{ V} \quad (4.22)$$

From (4.9)

$$E_n = 110 - 0.14 \cdot 29.18 - 0.514 = 105.4 \text{ V} \quad (4.23)$$

e. Rated electromagnetic torque, T_{en}, is calculated from (4.6)

$$T_{en} = \frac{E_n \cdot I_n}{2\pi n} = \frac{105.4 \cdot 29.18}{2\pi \cdot \frac{1200}{60}} = 24.48 \text{ Nm} \quad (4.24)$$

f. The shaft torque, T_{load}, comes directly from rated power P_n and speed n_n

$$T_{load} = \frac{P_n}{2\pi n_n} = \frac{3000}{2\pi \cdot \frac{1200}{60}} = 23.88 \text{ Nm} \quad (4.25)$$

g. The ideal no-load speed, n_0, corresponds to zero armature current in the voltage equation (4.9)

$$V_{an} = K_e \cdot n_0 \cdot \lambda_p \quad (4.26)$$

but $K_e.\lambda_p$ is constant

$$K_e \cdot \lambda_p = \frac{E_n}{n_n} = \frac{105.4}{\frac{1200}{60}} = 5.27 \text{ Wb} \quad (4.27)$$

Consequently

$$n_0 = \frac{V_{an}}{K_e \cdot \lambda_p} = \frac{110}{5.27} = 20.8728 \text{ rps} = 1252.37 \text{ rpm} \quad (4.28)$$

h. For regenerative braking the armature current becomes negative $I_{gn} = -I_n$ also $\Delta V_b = -\Delta V_b$, E_n remains the same and thus, from (4.9),

$$V_{ag} = E_n - R_a \cdot i_a - \Delta V_b = 105.4 - 0.14 \cdot 29.18 - 0.514 = 100.8 \text{ V} \quad (4.29)$$

So the voltage produced by the PEC should be simply reduced below the e.m.f. E_n level to produce regenerative braking for a given speed. The operating point moves from A in the first quadrant to A′ in the second quadrant (Figure 4.5).

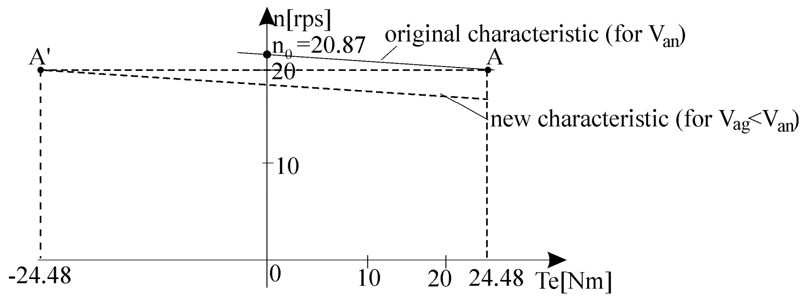


Figure 4.5. Torque/speed curves when switching from motoring to generating

4.6. VARYING THE SPEED

Example 4.2.

For the motor in example 4.1 and assuming that the mechanical losses are proportional to speed squared, while core losses depend, in addition, on flux squared, calculate:

a. the voltage V_a required and efficiency at $n_n / 2 = 600$ rpm for motoring at rated current;

b. for rated power and rated current at $2n_n = 2400$ rpm, calculate the flux weakening ratio and efficiency.

Solution:

a. From (4.21)

$$V_a = R_a \cdot i_a + E_n \cdot \frac{n}{n_n} + \Delta V_b =$$

$$0.14 \cdot 29.18 + 105.4 \cdot \frac{60}{1200} + 0.514 = 57.2992 \text{ V} \quad (4.30)$$

The mechanical and core losses are reduced 4 times (as the speed is halved) while the commutator losses remain the same as the current remains constant, I_n. So the total loss $\sum P$ is

$$\sum P = 0.04P_n + (0.01+0.01)\cdot P_n \cdot \left(\frac{1}{2}\right)^2 + (0.005+0.005)\cdot P_n$$
$$= (0.04+0.005+0.01)\cdot 3000 = 165\ \text{W} \tag{4.31}$$

The input power, P_{input}, is

$$P_{input} = V_a \cdot I_n = 57.2992\cdot 29.18 = 1672\ \text{W} \tag{4.32}$$

The efficiency η is

$$\eta = \frac{P_{input} - \sum P}{P_{input}} = \frac{1672-165}{1672} = 0.90 \tag{4.33}$$

It should be noted that even for half the rated speed, at full torque, the efficiency remains high. Consequently, varying speed through varying armature voltage is a high efficiency method.

b. When raising the speed above the base speed, at rated current and voltage, the e.m.f. remains the same as in (4.22): $E = E_n = 105.4$ V.

As the speed is doubled n = 2400 rpm the new flux level λ_p' is

$$\frac{\lambda_p'}{\lambda_p} = \frac{n_n}{n} = \frac{1200}{2400} = \frac{1}{2} \tag{4.34}$$

Thus the flux is halved and so is the electromagnetic torque T_e

$$\frac{T_e}{T_{en}} = \frac{\lambda_p'}{\lambda_p} = \frac{1}{2} \tag{4.35}$$

Now the losses, $\sum P'$, are

$$\sum P' = R_a \cdot I_n^2 + p_{add} + p_{iron} + p_{mec} + p_{com} =$$
$$= 0.14\cdot 29.18^2 + \left(0.005 + (0.01+0.01)\cdot\left(\frac{2400}{1200}\right)^2 + 0.005\right)\cdot 3000 = 300\text{W} \tag{4.36}$$

As the input power, P_{input} is still

$$P_{input} = V_n \cdot I_n = 110\cdot 29.18 = 3209.8\ \text{W} \tag{4.37}$$

The efficiency η is

$$\eta = \frac{P_{input} - \sum P'}{P_{input}} = \frac{3209.8-300}{3209.8} = 0.9065 \tag{4.38}$$

Again, the efficiency is high.
The above results are summarized in Figure 4.6.

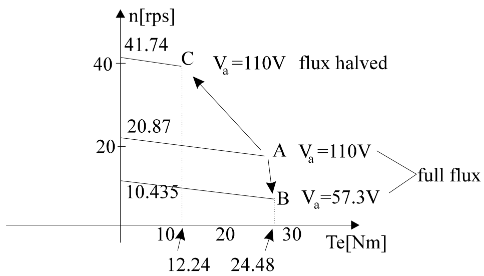


Figure 4.6. Torque/speed curves for varying speed from 1200 rpm (A) to 600 rpm (B) and, respectively, to 2400 rpm (C) for constant current.

Note that the electromagnetic power for points A and C is the same, while the electromagnetic torque for points A and B is the same. A 3 to 1 speed range (n_{max} / n_b) for constant power and current is quite feasible.

We may infer from here that below rated (base) speed the torque may be maintained constant for variable armature voltage while above rated (base) speed the electromagnetic power may be maintained constant through flux weakening at constant armature voltage as shown schematically in Figure 4.7.

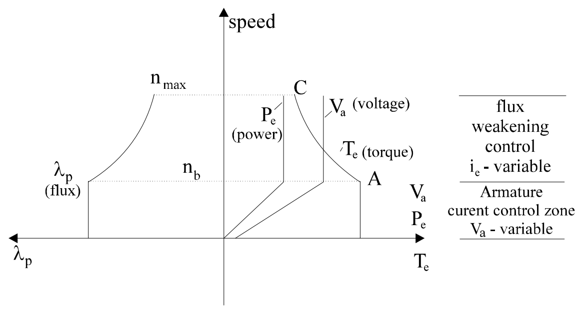


Figure 4.7. Speed/torque, power, voltage envelopes for constant current

Note: PM DC brush motors do not allow flux weakening and thus n_b = n_{max}.

There are applications with a constant power-wide speed zone (from n_b to n_{max}: 2-4n_b) such as in spindle drives where only electromagnetic excitation is adequate.

4.7. TRANSIENT OPERATION FOR CONSTANT FLUX

Constant flux means constant excitation (field) current or PM excitation. Consequently, in the performance equations (4.1)-(4.7), $di_e/dt = 0$ (λ_p = ct). Torque equation (4.6) becomes

$$T_e = \frac{K_e}{2\pi} \cdot \lambda_p \cdot I_a = K_T \cdot I_a \tag{4.39}$$

Similarly, the e.m.f. equation (4.1) is

$$E = K_e \cdot n; \quad K_T = \frac{K_e}{2\pi} \tag{4.40}$$

K_T is called the torque/current constant and in reality is constant only when the magnetic saturation level is constant or in the absence of magnetic saturation.

The variables, in performance equations, are now the armature current, i_a, and the speed, n. Equations (4.1)-(4.7) reduce to

$$\begin{aligned} V_a &= R_a \cdot i_a + L_a \cdot \frac{di_a}{dt} + K_e n \\ 2\pi J \cdot \frac{dn}{dt} &= \frac{K_e}{2\pi} \cdot i_a - T_{load} - B \cdot n \\ \frac{d\theta_r}{dt} &= 2\pi n \end{aligned} \tag{4.41}$$

As (4.41) constitutes a system of linear differential equations (no product of variables), Laplace transform may be used. For zero initial conditions we obtain

$$\begin{aligned} \tilde{V}_a &= (R_a + sL_a) \cdot \tilde{i}_a + K_e \tilde{n} \\ s\tilde{n} &= \frac{1}{2\pi J} \cdot \left(\frac{K_e}{2\pi} \cdot \tilde{i}_a - \tilde{T}_{load} - B \cdot \tilde{n} \right) \\ s\tilde{\theta}_r &= 2\pi \tilde{n} \end{aligned} \tag{4.42}$$

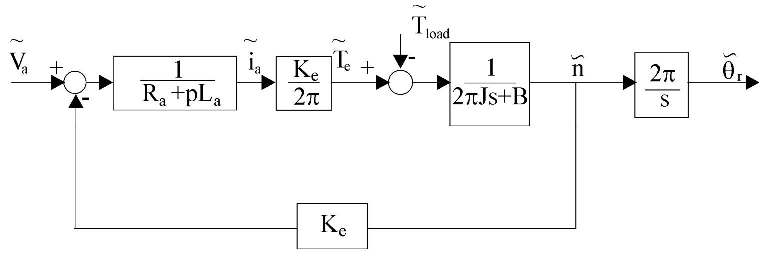


Figure 4.8. Constant flux block diagram of DC brush motor

Equation (4.42) suggests the block diagram of Figure 4.8, valid for small or large transients. So under constant flux (field current), the DC brush motor has two input variables $\tilde{V}_a$ and $\tilde{T}_{load}$ and three state variables $\tilde{i}_a$, $\tilde{n}$ and $\tilde{\theta}_r$. Only with a single input – either voltage $\tilde{V}_a$ for speed control ($\tilde{T}_{load}$ = 0) or load torque $\tilde{T}_{load}$ for torque control ($\tilde{V}_a$ = 0) – cascaded transfer functions can be obtained

$$\tilde{i}_a = \frac{\tilde{V}_a}{\left(R_a + sL_a\right) + K_e \cdot \dfrac{K_e}{2\pi} \cdot \dfrac{1}{2\pi Js + B}}; \tilde{n} = \frac{K_e \cdot \tilde{i}_a}{2\pi \cdot \left(2\pi Js + B\right)}; \tilde{T}_{load} = 0 \tag{4.43}$$

or

$$\tilde{i}_a = \frac{\tilde{T}_{load}}{\dfrac{K_e}{2\pi} + \dfrac{\left(2\pi Js + B\right)\cdot\left(R_a + sL_a\right)}{K_e}}; \tilde{n} = \frac{-\left(R_a + sL_a\right)\tilde{i}_a}{K_e}; \tilde{V}_a = 0 \tag{4.44}$$

The cascaded block diagrams described by (4.43)-(4.44) are shown in Figure 4.9.

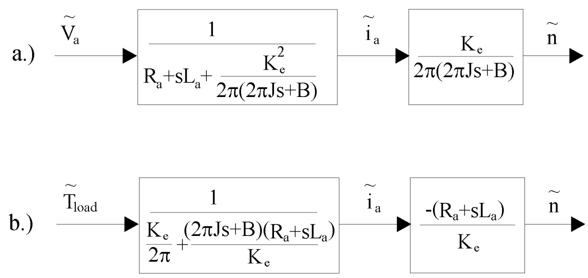


Figure 4.9. Cascaded transfer function for constant flux

a.) for speed control; b.) for torque control

The cascaded block diagrams are instrumental in designing the torque, speed or position control closed loops as shown in Chapter 7. The $\tilde{V}_a / \tilde{i}_a$ transfer functions have in both cases, Figure 4.9, the same poles (denominator's zeros) as a motor property

$$2\pi \cdot \left(2\pi Js + B\right) \cdot \left(R_a + sL_a\right) + K_e^{\,2} = 0 \tag{4.45}$$

Both poles $s_{1,2}$ have a negative real part, so the response is always stable. For B = 0 $s_{1,2}$ we have the simplified form

$$s_{1,2} = \frac{-1 \pm \sqrt{1 - 4\tau_e / \tau_{em}}}{2\tau_e} \tag{4.46}$$

with

$$\tau_e = \frac{L_a}{R_a}; \quad \tau_{em} = \frac{4\pi^2 J R_a}{K_e^{\ 2}} \tag{4.47}$$

where τ_e is the electrical time constant while τ_{em} is the electromechanical time constant. As seen from (4.46) for $4\tau_e \le \tau_{em}$, the response is aperiodically stable, while for $4\tau_e > \tau_{em}$ the response of the motor is periodically stable.

Low inertia fast response drives qualify for $4\tau_e > \tau_{em}$ and thus open-loop motor response is periodically stable.

4.8. PM DC BRUSH MOTOR TRANSIENTS

Example 4.3.

A PM DC brush motor has the following data: V_n = 110V, I_n = 10A, R_a = 0.5Ω, n_n = 1200rpm, τ_e = 2ms. For a 10V step voltage increase, calculate the speed response for J = 0.005kgm^2 and J′ = 0.05kgm^2, for constant load torque ($\tilde{T}_{load}$ = ct.) and B = 0 (no friction torque).

Solution:

From the voltage equation under steady state (4.20) we may calculate the e.m.f. E_n

$$E_n = V_n - R_a \cdot I_n = 110 - 0.5 \cdot 10 = 105 \text{ V} \tag{4.48}$$

From (4.40)

$$K_e = \frac{E_n}{n_n} = \frac{105}{\frac{1200}{60}} = 5.25 \text{ Wb} \tag{4.49}$$

Also from (4.40)

$$K_T = \frac{K_e}{2\pi} \tag{4.50}$$

The electromagnetic torque T_{em} (4.39) is

$$T_{em} = K_T \cdot I_n = 0.836 \cdot 10 = 8.36 \text{ Nm} \tag{4.51}$$

Under steady state, the load torque T_{load} is equal to the motor torque T_{em}.

In our case, it means that the initial and final current values are the same. Eliminating the current i_a in (4.41) we obtain

$$\tau_e \tau_{em} \frac{d^2 n}{dt^2} + \tau_{em} \frac{dn}{dt} + n = \frac{V_a}{K_e} - \frac{T_{load} \cdot R_a}{K_T \cdot K_e} \tag{4.52}$$

The roots of the characteristic equation of (4.52) are evidently equal to $s_{1,2}$ of (4.46). With $\tau_e = 2\text{ms}$

$$\tau_{em} = \frac{4\pi^2 J R_a}{K_e^2} = \begin{cases} \dfrac{4\pi^2 0.005 \cdot 0.5}{5.25^2} = 3.577 \cdot 10^{-3}\text{s} \\ \dfrac{4\pi^2 0.05 \cdot 0.5}{5.25^2} = 35.77 \cdot 10^{-3}\text{s} \end{cases} \tag{4.53}$$

Consequently (from 4.46)

$$s_{1,2} = \frac{-1 \pm \sqrt{1 - 4 \cdot 2 / 3.577}}{4 \cdot 10^{-3}} = -250 \pm j280; \quad \text{for } \tau_{em} = 3.577\text{ms}$$

and (4.54)

$$s_{1,2} = \frac{-1 \pm \sqrt{1 - 4 \cdot 2 / 35.77}}{4 \cdot 10^{-3}} = -250 \pm 218.8; \quad \text{for } \tau_{em} = 35.77\text{ms}$$

The final value of speed is obtained from (4.52) with zero derivatives

$$(n)_{t=\infty} = \frac{(V_a)_{t=\infty}}{K_e} - \frac{T_{load} \cdot R_a}{K_T K_e} = \frac{(110 + 10)}{5.25} - \frac{8.36 \cdot 0.5}{0.836 \cdot 5.25} = \\ = 21.904\text{rps} = 1314.27\text{ rpm} \tag{4.55}$$

The solution of (4.52) for the two cases is

$$n(t) = (n)_{t=\infty} + A \cdot e^{-250t} \cdot \cos(280t + \gamma); \quad \text{for } \tau_{em} = 3.577 \text{ ms}$$

and $$n(t) = (n)_{t=\infty} + A_1 \cdot e^{-468.8t} + A_2 \cdot e^{-31.2t}; \quad \text{for } \tau_{em} = 35.77 \text{ ms} \tag{4.56}$$

The initial conditions refer to the fact that

$$(n)_{t=0} = 20\text{rps}; \quad \left(\frac{dn}{dt}\right)_{t=0} = 0 \tag{4.57}$$

Finally,

$$A = 2.54; \quad \gamma = -41.76^0 \\ A_1 = 0.278; \quad A_2 = -2.046 \tag{4.58}$$

The two speed responses are drawn in Figure 4.10.

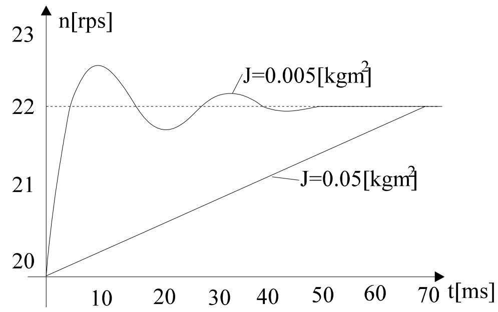


Figure 4.10. Speed responses to a step voltage increase from 110V to 120V at constant load torque

4.9. TRANSIENT OPERATION FOR VARIABLE FLUX

Variable flux means variable excitation current. This time the complete set of performance equations (4.1)-(4.7) has to be used. Even in the absence of magnetic saturation, when $\lambda_p = G \cdot i_e$, there are products of variables (i_e*i_a or i_e*n) which render the system nonlinear

$$\frac{di_e}{dt} = \frac{V_{ex} - R_e \cdot i_e}{L_e}$$
$$\frac{di_a}{dt} = \frac{V_a - R_a \cdot i_a - n \cdot G \cdot i_e}{L_a}$$
$$\frac{dn}{dt} = \frac{1}{2\pi J}\left(\frac{G \cdot i_e}{2\pi} i_a - T_{load} - B \cdot n\right) \tag{4.59}$$

The position θ_r is left out as position control is primarily performed with PM DC brush motor drives.

Through numerical methods, such as Runge Kutta-Gill, etc. the system (4.59) may be solved for large signal variables V_{ex}, V_a and T_{load}.

However, for the closed-loop control, design linearization around a steady-state point is standard

$$V_{ex} = V_{ex0} + \Delta V_{ex}; \quad V_a = V_{a0} + \Delta V_a; \quad i_a = i_{a0} + \Delta i_a;$$
$$i_{ex} = i_{ex0} + \Delta i_{ex}; \quad T_{load} = T_{load0} + \Delta T_l; \quad n = n_0 + \Delta n; \tag{4.60}$$

For the initial steady-state point, d/dt = 0 in (4.59)

$$V_{ex0} = R_e \cdot i_{e0}$$
$$V_{a0} = R_a \cdot i_{a0} + n_0 \cdot G \cdot i_{e0}$$
$$\frac{G \cdot i_{e0}}{2\pi} i_{a0} = T_{load0} + B \cdot n_0 \tag{4.61}$$

From (4.59) with (4.60)-(4.61) we get the matrix

$$\begin{vmatrix} \Delta V_{ex} \\ \Delta V_a \\ \Delta T_l \end{vmatrix} = \begin{vmatrix} R_e + sL_e & 0 & 0 \\ n_0 \cdot G & R_a + sL_a & G \cdot i_{e0} \\ \frac{G \cdot i_{a0}}{2\pi} & \frac{G \cdot i_{e0}}{2\pi} & -\left(J2\pi s + B\right) \end{vmatrix} \cdot \begin{vmatrix} \Delta i_e \\ \Delta i_a \\ \Delta n \end{vmatrix} \tag{4.62}$$

It is now evident, again, that the excitation circuit is decoupled from the armature circuit. The eigenvalues of matrix (4.62) are obtained by solving its determinant equation

$$\left(R_e + sL_e\right) \cdot \left(\left(R_a + sL_a\right)\left(J2\pi s + B\right) + \frac{G^2 \cdot {i_{e0}}^2}{2\pi}\right) = 0 \tag{4.63}$$

The first root of (4.63) s_0 is related to excitation

$$s_0 = -\frac{L_e}{R_e} \tag{4.64}$$

The other two roots $s_{1,2}$ are identical to those obtained for constant flux (4.43)-(4.44). The field current i_e, when varied through ΔV_{ex}, produces a notable delay in the current Δi_a and speed Δn response and thus should be avoided when fast transients are required. On the other hand, variable flux is useful for extending the speed/torque envelope for a given armature voltage V_{an}.

4.10 SPEED/EXCITATION VOLTAGE TRANSFER FUNCTION

Example 4.4.

For a DC brush motor with separate excitation, find the speed to excitation voltage transfer function based on the following numerical data: i_{e0} = 5A, R_e = 1Ω, L_e = 1H, R_a = 0.1Ω, L_a = 5mH, I_{a0} = 100A, J = 1Kgm^2, n_0 = 1200rpm, B = 0, V_{a0} = 210V.

Solution:

The required transfer function may be obtained from (4.62), with ΔV_a = 0 and ΔT_l = 0, by eliminating Δi_e and Δi_a

$$\frac{dn}{dv_{ex}} = \frac{G \cdot i_{a0} \cdot \left(R_a + sL_a\right) - n_0 \cdot G^2 \cdot i_{e0}}{\left(R_e + sL_e\right) \cdot \left[G^2 \cdot {i_{e0}}^2 + 4\pi^2 Js \cdot \left(R_a + sL_a\right)\right]} \tag{4.65}$$

In (4.65) we have all the parameters with the exception of G

$$E_0 = V_{a0} - R_a \cdot i_{a0} = 210 - 0.1 \cdot 100 = 200 \text{ V} \tag{4.66}$$

$$G = \frac{E_0}{n \cdot i_{e0}} = \frac{200}{\frac{1200}{60} \cdot 5} = 2 \text{ H} \tag{4.67}$$

Finally,

$$\frac{dn}{dV_{ex}} = \frac{2 \cdot 100 \cdot (0.1 + 0.005s) - 20 \cdot 2^2 \cdot 5}{(1 + 1 \cdot s)(2^2 \cdot 5^2 + 4\pi^2 \cdot 1 \cdot s \cdot (0.1 + 0.005s))} = \frac{s - 380}{(1+s)(100 + 3.4438s + 0.1972s^2)} \tag{4.68}$$

4.11. THE DC BRUSH SERIES MOTOR

The DC brush series motor schematics is shown in Figure 4.11. The excitation and armature currents are equal to each other unless an additional resistance R_{ead} is connected in parallel to the excitation circuit to produce flux weakening.

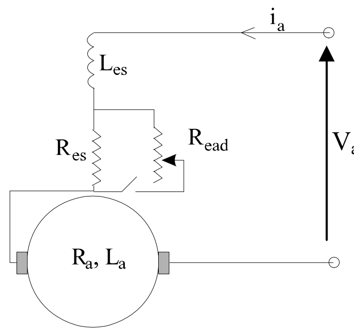


Figure 4.11. The DC brush series motor

With $R_{ead}=\infty$, the governing equations in terms of symbols shown in Figure 4.11 are

$$V_a = (R_a + R_{es}) \cdot i_a + (L_a + L_{es}) \frac{di_a}{dt} + nGi_a$$

$$T_e = \frac{Gi_a^2}{2\pi} = J2\pi \frac{dn}{dt} + T_{load} + B \cdot n \tag{4.69}$$

As it contains products of variables, the system (4.69) is nonlinear and thus solving it directly may be performed only through numerical methods.

For small perturbations, we may, however, linearize the equations

$$V_a = V_{a0} + \Delta V_a; \quad T_{load} = T_{load0} + \Delta T_l;$$
$$n = n_0 + \Delta n; \quad i_a = i_{a0} + \Delta i_a \tag{4.70}$$

to find

$$\begin{vmatrix} \Delta V_a \\ \Delta T_l \end{vmatrix} = \begin{vmatrix} R_a + R_{es} + n_0 \cdot G + s(L_a + L_{es}) & Gi_{a0} \\ 2\dfrac{G}{2\pi} i_{a0} & -(2\pi Js + B) \end{vmatrix} \cdot \begin{vmatrix} \Delta i_a \\ \Delta n \end{vmatrix} \tag{4.71}$$

The determinant of (4.71) leads to the eigenvalues of the system

$$(R_a + R_{es} + n_0 \cdot G + s \cdot (L_a + L_{es})) \cdot (2\pi Js + B) + 2\frac{G^2}{2\pi} \cdot i_{a0}{}^2 = 0 \tag{4.72}$$

The apparent electrical time constant in (4.72), if compared to the constant flux case (4.43), depends on speed through the term n_0G. At zero speed as $L_{es}>L_a$, and $R_{es}<R_a$, the electrical time constant is again larger than for the constant flux case when $\tau_e = L_a / R_a$.

Fast torque response is thus not easily expected from the DC brush series motor. However, it does not require a separate source to supply the field current, and flux weakening is simply possible through the shunt resistor R_{ead} (Figure 4.11).

For steady state, the speed/torque curve may be obtained from (4.69) with d/dt = 0.

$$i_a = \frac{V_a}{(R_a + R_{es}) + n \cdot G} = \sqrt{\frac{2\pi}{G} T_e} \tag{4.73}$$

At zero current (torque) the speed is infinite and thus the speed/torque curve is called mild (Figure 4.12).

Reducing the speed may be accomplished through V_a reduction, as done with DC brush motors with separate excitation.

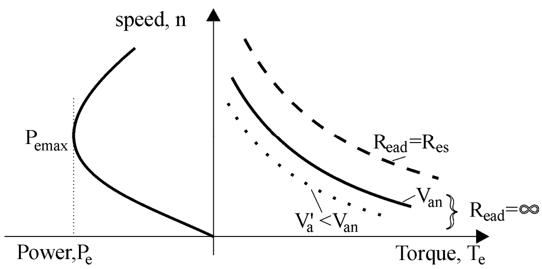


Figure 4.12. Speed/torque curve of DC brush series motor

Flux weakening may be performed through R_{ead} to obtain speeds above the rated (base) value for full armature voltage V_{an}. The electromagnetic power P_e is

$$P_e = T_e \cdot 2\pi n = \frac{V_a^2 \cdot G \cdot n}{\left(R_a + R_{es} + nG\right)^2} \tag{4.74}$$

Over a certain speed range electromagnetic power does not vary much and is, anyway, limited. This is a unique characteristic of the DC brush series motor that is so beneficial in transportation applications where the on-board installed power level is limited.

4.12. THE AC BRUSH SERIES MOTOR

Known also as the universal motor, the AC brush series motor is still used extensively in some home appliances such as washing machines, kitchen robots and vacuum cleaners. It is also predominant in hand-held (portable) tools and is fabricated up to 30.000 rpm at 1kW, in general. Despite the brush-burden the universal motor survived due to its low cost/performance.

Topologically is very similar to the DC brush series motor (Fig. 4.13) but the stator is made of stamped laminations because the stator excitation coils are AC fed. The commutation of the AC current at brushes is more difficult in comparison with DC brush motors, due to the AC transformer–type (speed independent)–additional–e.m.f. induced in the commutating coil in the rotor by the stator excitation AC current.

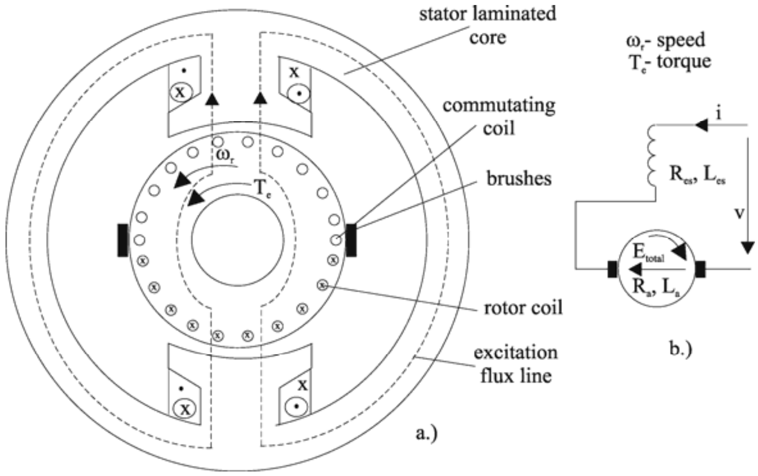


Figure 4.13. The two pole AC brush series motor

a) cross section; b) equivalent scheme

Because the stator excitation coils are series connected to the brushes, the field current and rotor current are equal to each other.

Also, as at brushes and in the stator, there is AC current; phasors may be used to investigate the machine steady state, despite the fact that, in the rotor, the current has two frequencies $\omega_r \pm \omega_1$ (ω_1 – stator frequency, $\omega_r = 2\pi np$ – electric rotor angular speed).

The voltage equation, valid also for transients, is straightforward:

$$(R_a + R_{es}) \cdot i - v = E_{pulse} - E_{rot} \tag{4.75}$$

E_{pulse} is the transformer (self induced) voltage in the stator and rotor inductances L_{es} and L_a:

$$E_{pulse} = -(L_{es} + L_a)\frac{di}{dt} \tag{4.76}$$

The rotational induced voltage (the motion e.m.f.) E_{rot} is, as for the DC brush machine, in phase with the stator current:

$$E_{rot} = k_e k_\Phi n i \tag{4.77}$$

It is evident that only the motion e.m.f. produces electromagnetic power P_{elm}:

$$P_{elm} = E_{rot} i = T_e 2\pi n \tag{4.78}$$

So the instantaneous torque T_e (from 4.78 with 4.77) is:

$$T_e = k_e k_\Phi \frac{i^2}{2\pi} \tag{4.79}$$

Now for steady state the stator voltage and current are sinusoidal:

$$v = V\sqrt{2}\cos\omega_1 t; i = I\sqrt{2}\cos(\omega_1 t - \varphi_1); \tag{4.80}$$

Consequently with I from (4.80), the instantaneous torque T_e is:

$$T_e = \frac{k_e k_\Phi I^2}{2\pi}\left[1 - \cos(2\omega_1 t - \varphi_1)\right] \tag{4.81}$$

So the instantaneous torque, for steady state, has a constant component which is the average torque and an AC (pulsating) component at $2\omega_1$ (Fig. 4.14a). The $2\omega_1$ torque pulsations mean vibrations, noise, and also additional losses.

For steady state from now on, we make use of phasors concept:

$$\underline{V} = V\sqrt{2} \cdot e^{j\omega_1 t}; \underline{I} = I\sqrt{2} \cdot e^{j(\omega_1 t - \varphi_1)} \tag{4.82}$$

With this denomination, the voltage equation (4.75) turns into:

$$\underline{V} = (R_{es} + R_a)\underline{I} + j\omega_1(L_{es} + L_a)\underline{I} + k_e k_\Phi n \underline{I} \tag{4.83}$$

$$R_{ae} = R_a + R_{es}; \omega_1(L_a + L_{es}) = X_{ae}$$

We may also include the stator and rotor core losses by replacing the total reactance X_{ae} to a series impedance Z_{ae}:

$$\underline{Z}_{ae} \approx R_{core} + jX_{ae} \tag{4.84}$$

Consequently, with core losses included, voltage eqn. (4.83) becomes:

$$\underline{V} = (\underline{R}_{ae} + \underline{Z}_{ae} + k_e k_\Phi n)\underline{I} \tag{4.85}$$

The average torque is now evident:

$$T_{eav} = \frac{(k_e k_\Phi n \cdot I) \cdot I^*}{2\pi n} = \frac{k_e k_\Phi I^2}{2\pi} \tag{4.86}$$

The dependence of torque on speed n is very similar to the case of the DC brush series motor (Fig. 4.14b) but with the torque reduced by the presence of total reactance X_{ae}.

Also the power factor $\cos\varphi_1$ is:

$$\cos\varphi_1 = \frac{1}{\sqrt{\left(\frac{X_{ae}}{R_{ae} + k_e k_\Phi n + R_{core}}\right)^2 + 1}} \tag{4.87}$$

The larger the speed n (or the np/f_1 ratio) the larger the power factor (Fig. 4.14b). A ratio $np/f_1 > 6 / 1$ is common to vacuum cleaners and for kitchen robots with only 3 / 1 in washing machines.

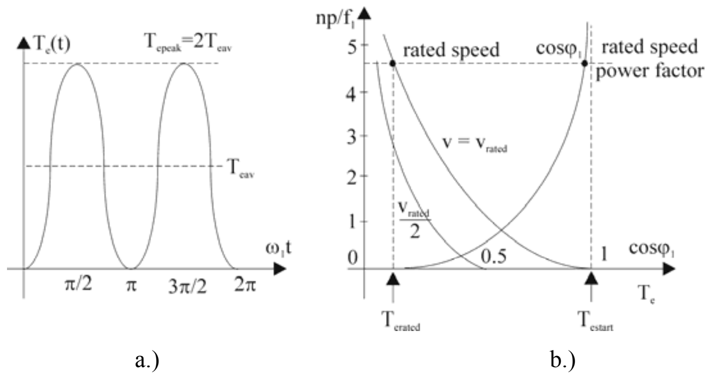


Figure 4.14. AC brush series motor steady state characteristics:

a) instantaneous torque, b) torque and power factor versus relative speed np / f_1

In general the universal motor is designed for a power factor above 0.9 for rated speed, which, in low power drives is a good asset in the absence of capacitors (required for the AC brushless motors). For a comprehensive analysis see Ref. 5.

Speed control may be approached simply by an AC-AC voltage changer that makes use of a single bidirectional thyristor module (called Triac). Torque equation (4.86) illustrates its quadratic dependence on the voltage amplitude.

The Triac is a very low cost device with simple controllability but it produces notable line current harmonics which have to be filtered. Also, when the voltage is decreased, the displacement power factor (DPF) is reduced.

Example 4.5.

Let us consider a universal motor for a home appliance supplied at 120 V AC (RMS), 60 Hz, that produces P_n = 600W at 18000rpm. It has two poles. The core loss is equal to half the copper loss and the mechanical loss is 2% of rated power. The efficiency is $\eta = 0.9$ and the power factor $\cos\varphi_n$ = 0.97. Calculate:

a) The rated current;
b) The total winding resistance R_{ae} and the core loss resistance R_{core};
c) The motion induced e.m.f. E_{rot};
d) The total inductance L_{ae};
e) The electromagnetic average torque;
f) The shaft torque;
g) The ratio of speed to frequency $n_n p/f_1$;
h) The starting current and torque;
i) The starting current when AC fed at 120 V DC

Solution:

a) From the definition of efficiency as output to input power:

$$\eta_n = \frac{P_n}{V_n I_n \cos\varphi_n}$$

The rated current I_n is:

$$I_n = \frac{600}{120 \cdot 0.97 \cdot 0.9} = 5.7273\text{A}$$

b) The total losses in the machine $\sum p$ are:

$$\Sigma p = \frac{P_n}{\eta_n} - P_n = 66.66\text{W}$$

With mechanical losses:

$$p_{mec}=0.02\ P_n=12\text{W},$$

The winding plus iron losses are:

$$p_{copper} + p_{core} = R_{ae} I_n^2 + R_{core} I_n^2 = \Sigma p - p_{mec} = 66.66 - 12 = 54.66 W$$

With the core losses as half the winding losses:

$$R_{ae} I_n^2 = 54.66; R_{ae} = \frac{54.66 \cdot 2}{3} \frac{1}{5.7273^2} = 1.111 \Omega$$

$$R_{core} = \frac{1}{2} R_{ae} \approx 0.555 \Omega$$

From (4.87)

$$I_n = \frac{U_n \cos \varphi_n}{R_{ae} + R_{core} + k_e k_\Phi n}$$

$$\tan \varphi_n = \frac{X_{ae}}{R_{ae} + R_{core} + k_e k_\Phi n}$$

So:

$$R_{ae} + R_{core} + k_e k_\Phi n = \frac{120 \cdot 0.97}{5.7273} = 20.3237 \Omega$$

The motion induced e.m.f.:

$$E_{rot} = k_e k_\Phi n I_n = (20.3237 - 1.111 - 0.555) \cdot 5.7273 = 106.858 V$$

Finally the machine reactance X_{ae}, from the impedance definition is:

$$X_{ae} = \frac{V_n}{I_n} \sin \varphi_n = \frac{120}{5.7273} 0.243 = 5.0936 \ \Omega$$

So the machine inductance:

$$L_{ae} = \frac{X_{ae}}{\omega_1} = \frac{5.0936}{2\pi 60} = 0.0135 H$$

e) The average torque T_{eav} comes from (4.79):

$$T_{eav} = \frac{k_e k_\Phi I_n^2}{2\pi} = \frac{E_{rot} I_n}{2\pi \cdot n_n} = 0.3248 \ \text{Nm}$$

f) The shaft torque T_{shaft} is approximately:

$$T_{shaft} \approx T_{eav} - \frac{p_{mec}}{2\pi n_n} = 0.3248 - \frac{12}{2\pi \cdot 300} = 0.31847 \ \text{Nm}$$

g) The ratio $n_n p/f_1 = (18000/60)/60 = 5/1$ explains the good power factor of the machine.

h) The starting current is obtained again from (4.85) but with n=0:

$$I_{start} = \frac{V_1}{\sqrt{(R_{ae} + R_{core})^2 + X_{ae}^2}} = 22.39A$$

The average starting torque $(T_{eav})_{start}$ is (4.79):

$$(T_{eav})_{start} = T_{eav}\left(\frac{I_{start}}{I_n}\right)^2 = 4.9646 \text{ Nm}$$

Note: In reality for this high current the magnetic flux saturates heavily and thus the ideal starting torque above is reduced considerably.

i) For DC at start, there is no AC (pulse) voltage across the machine inductances (X_{ae}=0) and R_{core}=0 (no core losses in DC).

$$I_{startd.c.} = \frac{V_{dc}}{R_{ae}} = \frac{120}{1.111} = 108Ad.c.$$

So, the brush series motor is to be fed from a much smaller DC voltage if the starting current is to be limited to reasonable values (3 – 5 times the rated value).

Note: In conjunction with PECs, the DC brush series motor is still widely used in standard electric propulsion systems for urban, interurban or water transportation and some heavy duty off-highway vehicles. Though the AC drives are taking over the electric propulsion technologies, we felt it was useful to devote two pages to the DC brush series motor, the workhorse of electric propulsion in the 20th century; not in the 21st century, however. A similar fate faces the a.c brush series motor.

4.13. SUMMARY

- The DC brush motor may be electromagnetically or PM excited in the stator. For variable speed drives, due to current pulse additional losses for PEC control, both the stator and rotor cores are laminated.
- For the axial airgap disk-rotor PM d.c brush machine, the rotor windings are in air and thus the core losses in the machine are very small and the electrical time constant is small (≤1ms in the kW power range), allowing for fast current (torque) control.
- Separately excited or PM machines, for constant field current (or PM), are second-order linear systems and, due to the feedback of e.m.f., they provide stable responses in speed or current; either periodic or aperiodic.
- The variation of field current introduces an additional, large delay in the response, corresponding to the excitation circuit time constant. Still the responses in speed and current are stable.
- The inherent decoupling between excitation and armature windings allows for quick armature current (torque) control for constant field current.

- The DC brush series motor has a mild speed/torque curve while the one with separate excitation has a rigid, linear, speed/torque curve. The former also proved to be very useful in the electrical propulsion of various vehicles.
- The AC brush series (universal) motor still enjoys, sizeable markets in home appliance and hand-held tools at high speeds, due to low cost with simple speed control.

4.14. PROBLEMS

4.1. For the DC brush motor in example 4.1 add the fact that the rated field current is I_n = 1A and V_{excn} = 110 V, and determine:

   4.2. the voltage V_a at standstill for rated current;

   4.3. the field current at 3600 rpm and rated current, and the corresponding torque, and total input power.

4.4. For the PM DC brush motor in example 4.3, determine the time variation of armature current for the 10 V step in the armature voltage for constant load torque.

4.5. A DC brush series motor for a light rail urban transportation system has the data: V_{an} = 800 V (DC), P_n = 100 KW, n_n = 1200 rpm, rated efficiency η_n = 0.92, $p_{mec} = p_{iron}$ = 0.015 P_n, $R_{es} = R_a$. The commutator and additional losses are neglected. Calculate:

   4.6. the rated current, I_n;

   4.7. armature and excitation resistances, R_a and R_{es};

   4.8. the rated e.m.f., E_n, and electromagnetic torque, T_{en}.

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